Victim Alignment in Crosstalk Aware Timing Analysis

Ravikishore Gandikota, Kaviraj Chopra, David Blaauw, Dennis Sylvester, Murat Becer*, Joao Geada*
Department of EECS, University of Michigan, Ann Arbor, MI. email: {gravkis, kaviraj, blaauw, dennis}@eecs.umich.edu
*CLK Design Automation, Littleton, MA. email: {murat, joao}@clkda.com

Abstract

Modeling the effect of coupling noise on circuit delay is a key issue in static timing analysis (STA) and involves the "victim-aggressor alignment" problem. As delay-noise depends strongly on the skew between the victim-aggressor input transitions', it is not possible to apriori identify the victim input transition that results in the latest arrival time at the victim. Several approaches that heuristically search for the worst-case victim-aggressor alignment have been proposed in literature. In this paper we present an analytical result that obviates the need to search for the worst-case victim input transition, thereby simplifying the victim-aggressor alignment problem significantly. Using the properties of standard nonlinear CMOS drivers, we show that regardless of the switching of the aggressors, the worst-case victim input transition is the one that switches at the latest point in its timing window. Although this result has been empirically observed in the industry, to the best of our knowledge, this is the first work that provides a rigorous analysis and shows that the result holds for both linear and non-linear drivers. We also show that limiting the alignment of the victim to only the latest victim input transition can significantly reduce the runtime of existing heuristic techniques with no loss of accuracy.

1. Introduction

Capacitive-coupling noise has become an important issue when performing timing verification of physical designs. The switching characteristic of a net is affected by the simultaneous switching of nets which are in close physical proximity. The net under analysis is referred to as the *victim* and all neighboring nets are termed as *aggressors*. The coupling noise injected by an aggressor can either slowdown or speedup the victim transition depending on the mutual victim-aggressor switching directions. The change in victim *arrival time* (usually the time at which it crosses 50% of supply voltage) is referred to as *delay-noise*. It is important to quantify the maximum delay-noise while performing static timing analysis for sign-off.

Static coupling noise analysis was first introduced in [1] and since then it has been the focus of significant research efforts. As delay-noise requires the aggressor and victim nets to switch in close temporal proximity of each other, the concept of *timing windows* was developed which identify the interval of the clock period within which a net can transition. Consequently, we can ignore those aggressors whose timing windows do not overlap with the victim timing window. However, it was observed that the computation of delay-noise and timing windows is not mutually independent. Delay-noise cannot be computed before the timing windows are defined, and conversely timing windows cannot to be computed without any information about the delay-noise. However, in [2],[3],[4] it was shown that this *'chicken-and-egg'* problem can be solved using an iterative approach. The iterations start with either the assumption that all aggressor timing windows overlap with that of the victim, or that there is no overlap between the victim-aggressor timing windows. In each iteration, the worst-case victim-aggressor alignment is determined by updating the timing windows with delay noise computed in the previous iteration. Delay-noise is then recomputed and then timing windows are updated accordingly until the two converge. It was shown in [2] that this iterative method is guaranteed to converge and in [4] it was theoretically established that timing analysis with crosstalk is a fixpoint solution on a complete lattice.

A fast and accurate delay-noise computation engine is key since the delay-noise engine is present in the inner loop of noise analysis. We know that delay-noise is very sensitive to the skew between the aggressor and victim arrival times. Therefore, it is non-trivial to find the *worst-case alignment* between the aggressor and the victim transitions, such that the output arrival time of the noisy victim is maximized [5],[6]. For a better understanding of the problem, consider two transitions at the input of the victim, one switching earlier than the other (as shown in Figure 2). If the early victim output transition couples more strongly with an aggressor or aligns with additional aggressors, then its delay-noise can be larger as compared to that of the later victim transition. However, it is not clear if a greater delay-noise in the earlier transition can result in a later victim output arrival time. Hence, it is difficult to apriori determine which victim input arrival time will produce the latest victim output arrival time. Therefore, in order to determine the worst-case alignment of the victim transition, we must compute the maximum delay noise for both the victim transitions and then pick the one which results in the latest output arrival time.

Initial approaches for performing delay-noise analysis (see [2],[7],[8],[9]) used coupling factors (e.g. 0-2) to appropriately scale the coupling capacitance. As delay-noise depends on numerous factors: such as the victim-aggressor alignment, the slew rates and the drive strengths of the victim-aggressor pair, a simple scaling of the coupling capacitances does not provide adequate accuracy. A bruteforce solution to the victim-aggressor alignment problem can be obtained by sweeping the victim arrival time exhaustively within its timing window, finding the worst-case aggressor alignment for each victim transition and selecting the one which results in the latest victim output arrival time. Since an exhaustive sweep is not practical for large circuits, several heuristic methods have been proposed in [6],[10],[11],[12]. In [6], the authors formulate the alignment problem as a weighted channel density problem. An empirical model-based approach is used to predict the alignment in [10]. In [11] an *effective skew window* model is proposed which results in a pessimistic estimation of delay-noise. In [12] the concept of *effective delay-noise* was introduced to capture the maximum change in the victim timing window due to coupling noise. Recently, in [13] the authors solve the alignment problem as a constrained optimization problem by using nonlinear simulations for evaluating the nonlinear objective function.

All the approaches outlined above either present a heuristic or perform a computationally expensive search in the victim timing window to solve the victim-aggressor alignment problem. In con-

This work was supported in part by funding from the National Science Foundation (NSF)

trast to these, we present in this paper an analytical result that obviates the need to enumerate the victim transitions occurring within in its timing window. Using the properties of standard nonlinear CMOS drivers, we show that the latest output arrival time of a victim net occurs only when its input transition occurs at the *latest* point in its timing window. Since, we only need to compute the worst-case alignment of the aggressors, the alignment problem is now significantly simplified. This result has been empirically observed in the industry and is already used in certain industrial noise analysis tools as an efficient heuristic to avoid enumerating the victim timing window. However, to the best of our knowledge this is the first work which analytically shows that the above result is optimal for both linear and nonlinear driver models. While the proof is fairly straight-forward for linear driver models, it is non-trivial for nonlinear drivers which is the case of practical concern.

We know that delay-noise can decrease as the victim input arrival time is increased (Region B of Figure 1). However, we *analytically* show that this decrease in delay noise is always less than the shift in the victim input arrival time. Using this result, we can always align the victim input transition at its latest possible arrival time and compute the worst-case delay noise to obtain the latest victim output arrival time. This significantly reduces the complexity of the worst-case victim-aggressor alignment problem, since we no longer need to sweep the victim throughout its timing window. Furthermore, the total number of aggressors which couple noise to the latest occurring victim transition is less than those coupled to a victim transition that can occur at any point in its timing window. We also present results that demonstrate significant speedups over proposed heuristics without compromising accuracy.

2. Problem Description

The focus of this paper is to find the optimal alignment of the victim transition in delay-noise analysis. In particular, we want to solve for a victim *input* arrival time such that the delay-noise results in the latest output arrival time. The plot of the victim stage delay as a function of the difference between the input arrival time of the victim and the aggressor (referred to as the input skew) is shown in Figure 1. For large values of both positive and negative input skew, there is no temporal overlap between the victim-aggressor transitions and the victim delay remains unchanged. However, for smaller values of input skew the aggressor transition couples noise to the victim transition and affects its stage delay. In Region A of the plot, the victim stage delay increases with an increase in its input arrival time. This is due to an increase in the temporal overlap between the victim-aggressor transitions. Once the victim stage delay peaks any further increase in the victim input arrival time leads to a decrease in the stage delay (in Region B) due to the reduction in delay-noise.

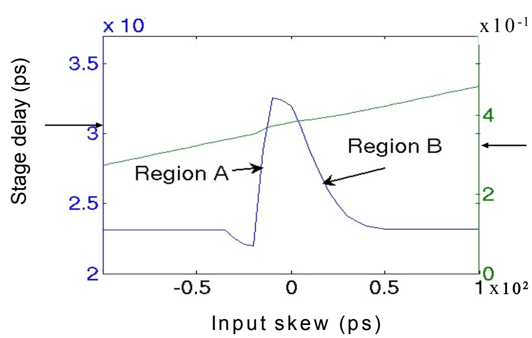

Figure 1. Delay-noise and output arrival time of a victim as a function of the skew between victim-aggressor input transitions

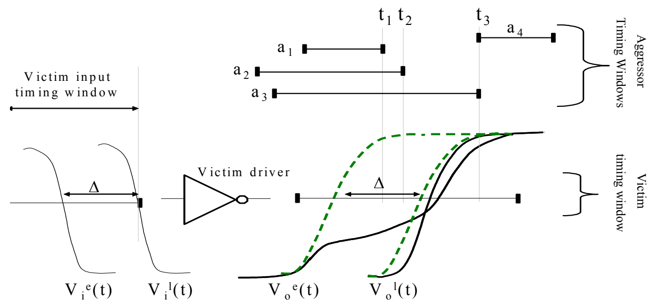

Figure 2. Victim alignment for worst-case delay and possible cross-over of noisy output transition.

Suppose the magnitude of the slope in Region B is always less than 1. In other words, the decrease in the stage delay in Region B is always less than the increase in its input arrival time. Since, the output arrival time is the sum of the input arrival time and the stage delay, the victim output arrival time will always be a monotonic increasing function of its input arrival time [11]. Therefore, the latest victim output arrival time would occur only when the victim input switches at the latest arrival time. However, it is not easy to show that the slope in Region B is always less than 1. It is especially difficult to show that the above holds for nonlinear drivers since the analysis is complicated by the cyclic nonlinear dependency between the aggressor and the victim responses [13].

For example, consider the aggressors a_{1-3} coupled to a victim net and their respective timing windows (as shown in Figure 2). Let $v_i^e(t)$ and $v_i^l(t)$ be two falling victim input transitions, where $v_i^e(t)$ switches earlier than $v_i^l(t)$ by an amount Δ. The noiseless output waveforms (dashed waveforms) must also be separated by Δ. It can be seen that the noisy output transition $v_o^e(t)$, corresponding to the early input transition $v_i^e(t)$, intersects with the timing windows of all three aggressors. However, when the victim input transition is delayed, the resulting output transition $v_o^l(t)$ can only couple with aggressor a_3. Consequently, the delay-noise observed for $v_o^e(t)$ is greater than that of $v_o^l(t)$. In such a case, if the difference between the delay-noise of $v_o^e(t)$ and $v_o^l(t)$ is greater than Δ, then the victim output waveforms will cross each other (as shown in Figure 2). Therefore, it is not clear whether the output arrival time of $v_o^l(t)$ will always be later than that of $v_o^e(t)$.

If we want to find the maximum victim output arrival time, we must allow any feasible victim transition occurring within its timing window. Furthermore, for each victim transition, we need to find the alignment of the aggressors such that delay-noise is maximized. Sweeping the victim transition and computing the aggressors alignment at each point is not practical, in particular for nonlinear driver models which require nonlinear simulations. As a result several heuristic solutions [11],[12] have been proposed to avoid an exhaustive search in the victim timing window. In [12], the authors proposed to enumerate victim alignment at the end points of aggressor switching windows. In Figure 2, for example, this would require analysis at three victim arrival times t_1, t_2, t_3.

In this work, we show that the magnitude of slope of the curve in Region B (of Figure 1) can never be greater than 1. This means that if the victim input is delayed and the delay-noise decreases due to misalignment, then this decrease is not sufficient to compensate for the fact that this victim output transition now starts later. This leads to the useful result that the worst-case victim alignment can occur only when the victim input is aligned at its latest arrival time. Consequently, even with nonlinear victim-aggressor drivers, any search of victim alignment within its timing window is not necessary. This significantly speeds up the noise analysis since we only need to find the worst-case aggressor alignment for the latest victim alignment. While this result has been empirically observed, to the best of our knowledge, this is the first work which proves that the above result holds for both linear and nonlinear driver models. Our proof is based on simple properties of standard, nonlinear CMOS drivers including complex gate drivers, which are discussed in Section 4.1. For simplicity in our analysis, we consider lumped interconnect loads and monotonic driver input transitions. Both the assumptions are discussed in more detail in Section 4.3.

The remainder of the paper is organized as follows: Section 3 proves the victim alignment result assuming linear driver models. Section 4 forms the core of this paper, where we prove this result for the more general case of nonlinear drivers. Runtime results shown in Section 5 confirms the efficacy of the proposed approach and in Section 6 we states our conclusions.

3. Victim alignment for a linear driver model

It is well-known that nonlinear driver models ([13],[14],[15]) give better accuracy in timing analysis than linear driver models. Nevertheless, linear driver models are still being used in existing industrial tools [16] for fast analysis in the early stages of design. For linear driver models, superposition principle can be used to break the cyclic dependency between the aggressor and victim responses. This simplifies the analysis for finding the worst-case victim-aggressor alignment. In this section we prove that for linear drivers, the latest victim output arrival time occurs only when the victim input transition is aligned at the latest arrival time. We will later review the victim alignment for non-linear driver models in Section 4.

Theorem 1. Given linear victim-aggressor driver models, the victim output transition obtained by aligning the victim input transition at the latest input arrival time bounds all possible victim output transition.

***Proof*:** Consider the victim-aggressor configuration as shown in Figure 3. Let $v_i^l(t)$ be the late victim input transition that is aligned at the latest arrival time in its timing window and $v_i^e(t)$ be any earlier input transition occurring anywhere within its timing window. The corresponding victim output transitions are denoted by $v_o^e(t)$ and $v_o^l(t)$ respectively. Without loss of generality, we assume rising victim and falling aggressor output transitions. Our goal is to show that the latest victim output transition $v_o^l(t)$ bounds any earlier victim output transition $v_o^e(t)$, or mathematically

$$v_o^e(t) \geq v_o^l(t) \forall t . \qquad (1)$$

It is necessary to show that Equation (1) holds for all feasible aggressor transitions. Therefore, we arbitrarily select an aggressor input transition which can occur anywhere within its timing window. Due to coupling noise, the aggressor output transitions $a_o^e(t)$ and $a_o^l(t)$, corresponding to the early and late victim transitions, may be different even though the input transition is the same (that is $a_i^e(t) = a_i^l(t)$). Applying the principle of superposition which

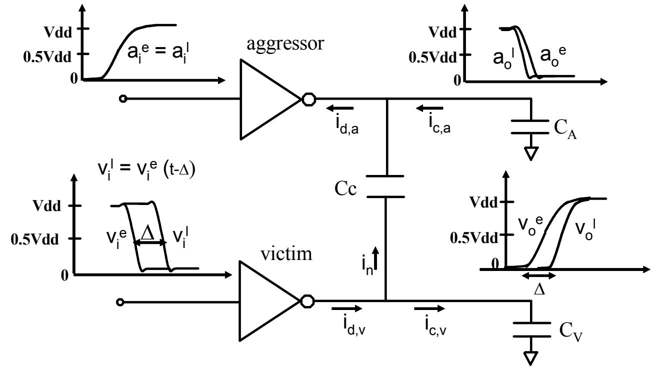

Figure 3. A coupled victim-aggressor configuration

holds for linear driver models, the noisy victim output transition $v_o^l(t)$ can be written as

$$v_o^l(t) = \bar{v}_o^l(t) + v_n(t) , \qquad (2)$$

where $\bar{v}_o^l(t)$ is the noiseless victim output transition obtained with a quiet aggressor and $v_n(t)$ is the noise waveform coupled to a quiet victim. Since the noise $v_n(t)$ remains the same in both (early/late) cases, the early victim noisy output transition $v_o^e(t)$ is given by

$$v_o^e(t) = \bar{v}_o^e(t) + v_n(t) . \qquad (3)$$

If the separation in time between the victim input transitions $v_i^e(t)$ and $v_i^l(t)$ is Δ, then the noiseless output waveforms $\bar{v}_o^e(t)$ and $\bar{v}_o^l(t)$ would also be separated by Δ, that is

$$v_i^l(t) = v_i^e(t-\Delta) \Rightarrow \bar{v}_o^l(t) = \bar{v}_o^e(t-\Delta) . \qquad (4)$$

Since the inputs are falling monotonically, the noiseless output transitions must therefore be rising monotonically. As a result, the late noiseless output transition $\bar{v}_o^l(t)$ always bounds the early noiseless output transition $\bar{v}_o^e(t)$, that is

$$\bar{v}_o^e(t) \geq \bar{v}_o^l(t) \forall t . \qquad (5)$$

As the noise $v_n(t)$ remains the same since we assume linear drivers, the noisy output transition $v_o^l(t)$ bounds $v_o^e(t)$, that is

$$v_o^e(t) \geq v_o^l(t) \forall t . \qquad (6)$$

Since the late victim output transition $v_o^l(t)$ is always less than the early victim output transition $v_o^e(t)$, it will always cross the half (or any other) supply voltage point later than $v_o^e(t)$. Therefore, the latest victim output arrival time can occur only when the input transition is aligned at the latest input arrival time. □

4. Victim alignment for nonlinear drivers

To model nonlinearity of CMOS drivers in noise analysis, nonlinear driver models such as current source models ([13],[14],[15]) have recently been developed which provide much better accuracy than linear models. In this section, we show that the victim alignment result derived in the previous section also holds for nonlinear drivers. We begin this section by describing the CMOS driver current characteristics.

4.1 Properties of Nonlinear drivers

To derive the desired alignment property, we consider a nonlinear *inverting* CMOS driver where $i_d(t) = f(v_i(t), v_o(t))$ is the current flowing *out* of the driver, $v_i(t)$ and $v_o(t)$ are the respective input and output voltages of the driver. It can be seen that $i_d(t)$ is the difference between the drive current sourced by the pull-up network and that sunk by the pull-down network,

$$i_d(t) = i_{pullup} - i_{pulldown} \quad . \tag{7}$$

From CMOS transistor characteristics we know that given a constant drain-source voltage, the drain current I_{ds} is a monotonic increasing function of its gate voltage V_{gs}. Also, the V_{gs} of the pull-up and the pull-down networks of a driver depends only on the input voltage $v_i(t)$, that is

$$V_{gs(pullup)} \propto V_{dd} - v_i(t) \quad , \quad V_{gs(pulldown)} \propto v_i(t) \tag{8}$$

Now, a decrease in input voltage affects only the V_{gs} of the pull-up and pull-down network. From basic transistor current-voltage characteristics it follows that, for a *constant* output voltage, a decrease in input voltage results in an increase (decrease) in the pull-up (down) current. The output drive current $i_d(t)$ given by the difference between the pull-up and the pull-down current (7) also increases (decreases). It is easy to see that the above also hold for more complex gates or gates with skewed transistor stacks.

A similar analysis leads to the observation that given a constant input voltage, a decrease in output voltage leads to an increase (decrease) in the pull-up (down) current, resulting in an increase in the drive current. Since the property relies only on the monotone behavior of I_{ds} with V_{ds} for MOSFET transistors, it also holds for complex gates with transistor stacks and internal nodes.

We sum up the above observations in the following property which relates the driver output current to driver input and output voltages:

Property 1. Given a particular input and output voltage, the magnitude of the drive current $i_d(t)$ flowing *out* of a CMOS driver *increases* with a *decrease* in its input or output voltage.

Strictly speaking, the above properties may not hold true during the entire transition due to the effect of Miller capacitance, especially if the driver input transitions very rapidly. The amount of Miller current strongly depends on the ratio of the Miller capacitance to the output load capacitance. However, delay noise is significant only for those victim nets that are coupled to several aggressors and which therefore have with substantial output loading. For such victim nets, the Miller current is typically negligible when compared to driver current and Property 1 implicitly holds. Furthermore, the Miller current is significant only for very fast input transitions and affects the initial part of the output transition which is of lesser of interest in noise analysis. We will now employ the property of driver current in our forthcoming analysis.

4.2 Worst-case victim alignment

In this section, we prove by contradiction that any early victim output transition can never cross any later victim output transition.

Theorem 2. Given nonlinear victim-aggressor drivers, the victim output transition obtained by aligning the victim input transition at the latest input arrival time, bounds any other feasible victim output transition.

***Proof*:** Consider the victim-aggressor configuration shown in Figure 3, where C_v and C_a denote the output load capacitances of victim and aggressor drivers respectively and C_c denotes the coupling capacitance. The victim input transition $v_i^l(t)$ is aligned at the latest time point in its timing window and $v_i^e(t)$ is an arbitrary earlier victim input transition. The corresponding victim output transitions are denoted by $v_o^e(t)$ and $v_o^l(t)$ respectively. Our goal is to show that the latest victim output transition $v_o^l(t)$ bounds any early victim output transition $v_o^e(t)$, expressed mathematically in (1). Without loss of generality, we assume rising victim and falling aggressor output transitions. Since, it is necessary to show that (1) holds for all feasible aggressor transitions, we arbitrarily select an aggressor input transition which can occur anywhere within its timing window. Due to coupling noise, the aggressor output transitions $a_o^e(t)$ and $a_o^l(t)$, corresponding to the early and late victim transitions, may be different even though the input transition is the same (that is $a_i^e(t) = a_i^l(t)$). We first present an outline of the proof:

1. **Victim response analysis**: Suppose a later output transition crosses an early output transition. Then, at the cross over point, we obtain a necessary relationship between the corresponding *noise currents* by analyzing the rate of change of the victim output response.
2. **Aggressor response analysis**: Next, using this relationship between noise current and the fact that aggressor input transition is same for both the cases, we compare the relative magnitudes of aggressor driver currents and derive a necessary relationship between the *aggressor output* responses.
3. **Charge Conservation**. We then analyze the charge accumulated across the coupling capacitance due to both the early and late victim transitions and show that the necessary relationship between aggressor output responses cannot be satisfied.

We prove by **contradiction** that the later victim output transition $v_o^l(t)$ must always bound any earlier victim output transition $v_o^e(t)$, or in other words $v_o^l(t) \le v_o^e(t) \forall t$.

Victim response analysis: We begin our proof by analyzing the response at the output of the victim driver. Suppose, the converse is true and there exists a time τ_v when both the victim output waveforms cross each other for the *first* time (as shown in Figure 4),

$$v_o^l(t) = v_o^e(t)\big|_{t = t_v} . \tag{9}$$

From definition, the victim output transition $v_o^l(t)$ starts rising after $v_o^e(t)$. Therefore, if $v_o^l(t)$ manages to cross $v_o^e(t)$ at time τ_v, then it means that $v_o^l(t)$ must be rising at a faster rate than $v_o^e(t)$ at the time instant τ_v, that is

$$\frac{\partial}{\partial t}\{v_o^l(t)\} > \frac{\partial}{\partial t}\{v_o^e(t)\}\bigg|_{t = \tau_v} . \tag{10}$$

We know that the charging current flowing into the victim load C_v is given by $i_{c,v}(t) = C_v \times \frac{\partial}{\partial t}\{v_o(t)\}$. From (10) we obtain the following relationship,

$$i_{c,v}^l(t) > i_{c,v}^e(t)\big|_{t = \tau_v} , \tag{11}$$

where $i_{c,v}^e(t)$ and $i_{c,v}^l(t)$ are the currents flowing into the victim load corresponding to the two victim transitions $v_o^e(t)$ and $v_o^l(t)$ respectively (see Figure 3). At crossover time τ_v, the output voltages of the victim driver is equal. Under the assumption of monotonic falling victim input transitions, we also have the inequality $v_i^e(t) \le v_i^l(t)|_{t = \tau_v}$. It follows from Property 1 that the victim driver sources a larger current in the case of an early victim transition as compared to the late victim transition,

$$i_{d,v}^e(t) > i_{d,v}^l(t)|_{t = \tau_v}. \qquad (12)$$

Subtracting (11) from (12), we obtain

$$i_{d,v}^e(t) - i_{c,v}^e(t) > i_{d,v}^l(t) - i_{c,v}^l(t)\Big|_{t = \tau_v}. \qquad (13)$$

Applying Kirchhoff's Current Law (K.C.L.) at the victim output node and rewriting (13) in terms of the corresponding noise currents flowing through the coupling capacitance C_c, we obtain

$$i_n^e(t) > i_n^l(t)|_{t = \tau_v}. \qquad (14)$$

Aggressor response analysis: Using the information about the victim output transitions, we obtain a relationship between the early and late aggressor output waveforms. The noise current flowing through the coupling capacitance C_c can be expressed in terms of the rate of change of the voltage difference across its terminals. After plugging the expressions for noise current it into equation (14), we obtain

$$\frac{\partial}{\partial t}\{v_o^e(t) - a_o^e(t)\} > \frac{\partial}{\partial t}\{v_o^l(t) - a_o^l(t)\}\Big|_{t = \tau_v}, \qquad (15)$$

where $a_o^e(t)$ and $a_o^l(t)$ are the corresponding early and late aggressor output waveforms (see Figure 3). Rearranging both sides of equation (15), we obtain

$$\frac{\partial}{\partial t}\{a_o^l(t) - a_o^e(t)\} > \frac{\partial}{\partial t}\{v_o^l(t) - v_o^e(t)\}\Big|_{t = \tau_v}. \qquad (16)$$

From (10) we know that, at crossover time τ_v, the rate of change of the late victim transition $v_o^l(t)$ is greater than that of $v_o^e(t)$. Therefore, the term on the right hand side of (16) must be greater than zero and we get the following inequality between the early and the late aggressor output waveforms,

$$\frac{\partial}{\partial t}\{a_o^l(t)\} > \frac{\partial}{\partial t}\{a_o^e(t)\}\Big|_{t = \tau_v}. \qquad (17)$$

We now provide an *intuition* for the analytical results that have been obtained so far. Suppose, $v_o^l(t)$ rises at a relatively faster rate and crosses $v_o^e(t)$ at time τ_v. This implies two things about the relative current magnitudes of the currents flowing at the time instant τ_v. For the later victim output transition $v_o^l(t)$ (a) the current sourced by the victim driver is less, and (b) the charging current flowing into the interconnect load is more, as compared to the early victim transition $v_o^e(t)$. Since, the current sourced from the victim driver equals the sum of the noise current and the charging load current, at time τ_v, the noise current must be less for the later victim transition. Also, the noise current depends on the rate of change of the voltage difference across the coupling capacitance. At time τ_v, the rate of change of the later victim output transition is more. Therefore, the relationship among the noise currents demands that the rate of change of $a_o^l(t)$ be *less* than $a_o^e(t)$. Note that the aggressor has falling output transitions. Therefore, both the derivative terms in (17) are negative in magnitude and early aggressor $a_o^e(t)$ falls more rapidly than $a_o^l(t)$.

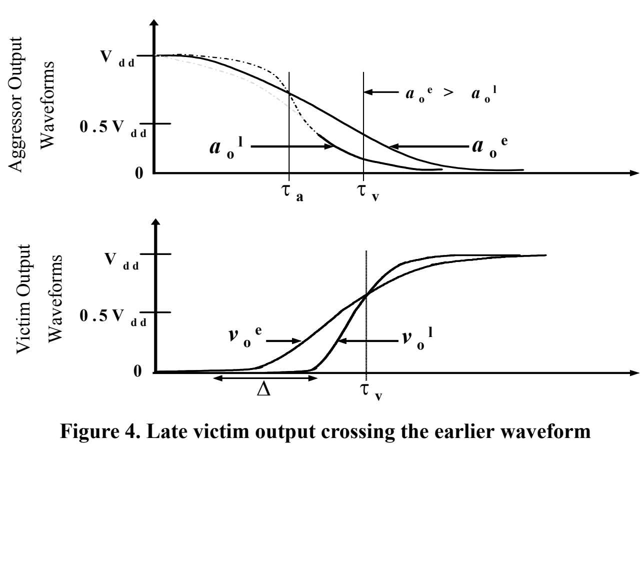

Figure 4. Late victim output crossing the earlier waveform

The discharging current of the aggressor interconnect load C_a is given by $i_{c,a} = -C_a \times \frac{\partial}{\partial t}\{a_o(t)\}$. The negative sign in the above expression is due to the convention followed that the load current is flowing out of the load capacitance C_a (see Figure 3). From (17), we obtain the following inequality

$$i_{c,a}^e(t) > i_{c,a}^l(t)|_{t = \tau_v}. \qquad (18)$$

Adding (18) and (14) together and applying K.C.L. at the aggressor output node, we obtain the following inequality among the aggressor driver currents,

$$i_{d,a}^e(t) > i_{d,a}^l(t)|_{t = \tau_v}. \qquad (19)$$

Since the aggressor input transitions are the same in both cases, that is $a_i^e(t) = a_i^l(t)$, the gate voltages of the aggressor driver are equal. Now, if the aggressor driver currents differ according to (19), then from Property 1 it follows that the output drain voltages of the aggressor must have the following relationship,

$$a_o^e(t) > a_o^l(t)|_{t = \tau_v}. \qquad (20)$$

To summarize, we obtain two necessary conditions on the aggressor output waveforms at time τ_v, that is $a_o^l(t)$ must be lower (20) and must transition at a slower rate than $a_o^e(t)$ (17).

Charge conservation analysis: At this point we will analyze the relationships between the driver current that is sunk by the aggressor driver in both cases. To do that, we need to define a time interval during which we compute the amount of charge sunk by aggressor driver. It follows from (20) that there must be a time $\tau_a : 0 \le \tau_a < \tau_v$ where the early and late aggressor output transitions intersect each other (see Figure 4),

$$a_o^l(t) = a_o^e(t)|_{t = \tau_a}. \qquad (21)$$

If the late aggressor output transition $a_o^l(t)$ is always less than $a_o^e(t)$, then $\tau_a = 0$ (as shown by the dotted waveform). If $a_o^l(t)$ and $a_o^e(t)$ have multiple crossovers before time τ_v, then we choose τ_a to be the time at which the *latest* crossover occurs between the aggressor output transitions. The boundary conditions on the victim-aggressor output transitions in the interval $t \in (\tau_a, \tau_v)$ are as follows (as shown in Figure 4),

$$a_o^e(\tau_a) = a_o^l(\tau_a) \quad , \quad a_o^e(\tau_v) > a_o^l(t) ,$$
$$v_o^e(\tau_a) > v_o^l(\tau_a) \quad , \quad v_o^e(\tau_v) = v_o^l(\tau_v) . \tag{22}$$

Since we chose τ_a to be the latest occurring crossover time of aggressor output transitions before τ_v, we obtain the following monotonic relationship between the aggressor output transitions

$$a_o^e(t) > a_o^l(t) \forall t \in (\tau_a, \tau_v) . \tag{23}$$

Recall that the input to the aggressor driver is the same in both cases. From Property 1 and equation (23), it follows that the current sunk by the aggressor driver in the time interval (τ_a, τ_v) is always greater in the early case,

$$i_{d,a}^e(t) > i_{d,a}^l(t) \forall t \in (\tau_a, \tau_v) . \tag{24}$$

Integrating the above, we obtain the inequality between the total charge sunk by the aggressor driver Q_d^e and Q_d^l respectively,

$$\int_{\tau_a}^{\tau_v} i_{d,a}^e(t) > \int_{\tau_a}^{\tau_v} i_{d,a}^l(t) \Rightarrow Q_d^e > Q_d^l . \tag{25}$$

We now analyze the relationships between the integrals of the noise current $i_n(t)$ and the load current $i_{v,a}(t)$ flowing into the aggressor in the time interval $t \in (\tau_a, \tau_v)$. As both integrals are state functions they do not depend on the integration path but only depend on the voltage values at the boundaries of the interval (τ_a, τ_v). The integral of load current $Q_{c,a}^e$ for the early victim transition is

$$Q_{c,a}^e = \int_{\tau_a}^{\tau_v} i_{c,a}^e(t) = -C_a \times \{a_o^e(\tau_v) - a_o^e(\tau_a)\} , \tag{26}$$

and similarly the integral of noise current Q_n^e is given by

$$Q_n^e = \int_{\tau_a}^{\tau_v} i_n^e(t) = C_c \times \{v_o^e(\tau_v) - v_o^e(\tau_a) + a_o^e(\tau_a) - a_o^e(\tau_v)\} . \tag{27}$$

Similarly, we can derive the integral of the load current ($Q_{c,a}^l$) and the integral of the noise current (Q_n^l) for the late victim transition. After plugging in the boundary values from (22), we obtain the following relationship,

$$Q_{c,a}^l > Q_{c,a}^e \text{ and } Q_n^l > Q_n^e . \tag{28}$$

Adding both inequalities in (28) and applying K.C.L on the aggressor output node, we obtain the following inequality

$$Q_d^l > Q_d^e \tag{29}$$

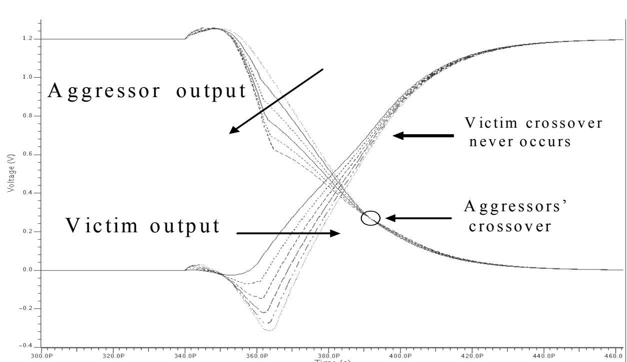

Figure 5. HSPICE simulation plots showing the victim-aggressor output transitions

which contradicts the necessary condition (25). This proves that victim output transition $v_o^e(t)$ can never cross $v_o^l(t)$ and the latest victim output arrival time occurs only when the victim input is aligned at its latest input arrival time □

To further illustrate the ideas put forward in the above proof, we show in Figure 5 the output responses generated in HSPICE for a 'pathological' victim-aggressor configuration. We fix the aggressor input arrival time and sweep the input skew by shifting the victim input arrival time to the right. Note that as we delay the victim input, the aggressor output initially starts to transition faster. This is because the Miller effect due to the switching of the victim is delayed since its starts switching later. Also, due to the noise coupled from the aggressor, the delayed victim output transition starts from a voltage below zero. This increases the drain-source voltage of the pull-up network of the victim driver and the victim output waveform starts rising rapidly. However, as the victim output transition starts approaching an earlier transition, the corresponding aggressor output transitions cross each other (see Figure 5). This violates the necessary condition of (20) and the victim output transitions never cross each other.

4.3 Assumptions

We now discuss the two assumptions made in our analysis:

- Lumped interconnect model: In this paper, for ease of understanding, the latest victim result is derived using a lumped interconnect load model. However, the desired contradiction in our proof is based on the conservation of the net charge flowing through the aggressor driver. To this end, we identified an interval (τ_a, τ_v) within which we compare the net charge flowing through the aggressor driver for the early and late cases. If we replace the lumped capacitive load with an RC load, then τ_v is chosen as the time at which the victim output transitions at the far end of the RC load (victim receiver input) cross each other for the first time. The time τ_a is chosen to be the latest crossover time, occurring before τ_v, between the aggressor output transitions at the near end of RC load (aggressor driver output). Performing a similar analysis, with these modified time intervals, it may be shown that all the relationships derived above will also hold true for a distributed RC network.
- Monotonic input transitions: With non-monotonicity in input waveforms, the latest victim input transition may no longer bound any earlier victim input transition. However, it is not clear whether this can actually result in a later victim output arrival time for an earlier victim input transition. Also, it should be noted that non-monotonic transitions filter out rapidly due to the low-pass filtering effects of CMOS drivers. Experimentally, we found that restricting our analysis to those input transitions that are monotonic did not significantly reduce the speedup that was obtained.

5. Results

A noise analysis tool was implemented in C++. The proposed approach of aligning the victim transition at the latest point in its timing window was implemented and its efficacy was tested on the MCNC benchmark circuits synthesized in 0.13nm technology. All experiments were run on a 1GHz SUN machine with 4GB of memory. The synthesized benchmarks were placed-and-routed by using a commercial APR tool. A commercial parasitic extraction tool was used to extract the distributed interconnect RC values and then noise analysis was performed using industrial timing libraries.

A summary of the experimental results obtained for MCNC benchmark circuits is listed in Table 1. The details of all circuits are given

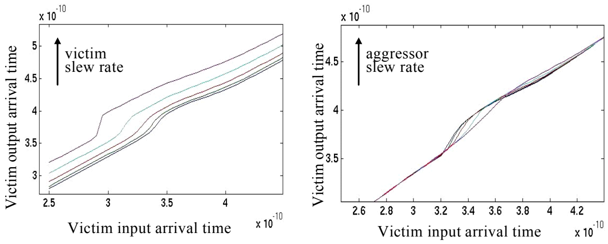

Figure 6. Plot of victim output vs. input arrival time by varying (1) victim slew rate, (2) aggressor slew rate

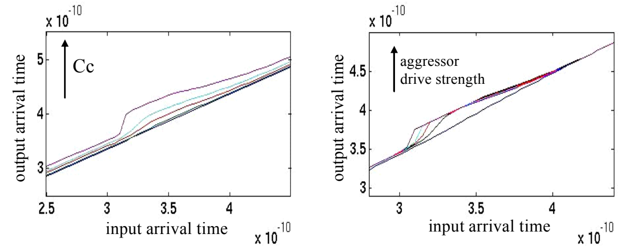

Figure 7. Plot of victim output vs. input arrival time by varying (1) coupling capacitance, and (2) aggressor drive strength

in the first four columns, while the results of the proposed approach are given in the last three columns. To obtain the worst-case victim output arrival time, we safely align the victim input transition at the latest victim input arrival time. Hence, there is no need to search within the victim timing window which leads to a further simplification in the alignment procedure. Consequently, all aggressors whose timing windows overlap with the victim timing window but do not overlap with the victim transition at the *latest point* in its timing window can be safely ignored in noise analysis. Table 1 shows that almost half of the total number of aggressors can be eliminated from delay-noise analysis in this manner. Furthermore, due to the fact that we no longer need to enumerate the victim timing window for computing the worst-case alignment, an average speedup of 4.3X was achieved over heuristic approaches [12] on benchmark circuits. Also note that the maximum speedup is achieved in circuit *i*9 which has the largest number of aggressors per victim net (~14). Hence, it is evident that larger speedups can be achieved for bigger industrial circuits.

Figure 6 shows the plots of the victim output arrival times obtained by sweeping the victim input arrival time of a typical victim-aggressor configuration, for different values victim-aggressor input slew rates. Figure 7 shows a similar plot, where we vary the driver strength and coupling capacitances of the victim-aggressor configuration. As expected, it can be seen that all plots are *monotonically* increasing, in agreement with our claim that the latest victim input transition results in maximum output arrival time.

6. Conclusion

In this paper, we prove that the latest victim output arrival time occurs only when its input transition is aligned at the latest point in its timing window. While the proof is fairly straight forward for linear drivers, it is non-trivial for non-linear CMOS drivers. The result in this paper obviates the need for enumerating the victim input timing window in delay-noise analysis. Consequently, the victim-aggressor alignment problem is simplified and its complexity is significantly reduced. Although this result has been observed empirically in the industry, this is the first work which analytically shows that the result holds for both linear and nonlinear drivers. We show that significant speedup can be achieved on benchmark circuits over existing heuristic solutions without incurring any loss of accuracy.

Table 1. Results for the proposed latest victim alignment

circuit name	# of nets.	# of agg	circuit delay (ns)	Proposed Approach		
				# of agg. pruned	run time(s)	speedup
i1	46	232	0.546	103(44%)	0.01	2.74
i2	221	706	0.743	324(46%)	0.02	2.46
i3	126	551	0.529	281(51%)	0.02	3.12
i4	230	1181	0.801	610(52%)	0.02	3.56
i5	138	1835	1.212	794(43%)	0.04	4.88
i6	668	7298	1.045	3066(42%)	0.14	5.15
i7	870	9605	1.124	4925(51%)	0.15	6.19
i8	1528	10235	1.636	5436(53%)	0.21	5.32
i9	955	14140	1.841	6789(48%)	0.33	6.91
i10	3155	18318	3.089	8744(48%)	0.45	3.21

References

[1] K. L. Shepard and V. Narayanan, “Noise in deep submicron digital design”, in *IEEE/ACM International Conference on Computer-Aided Design,* pages 524-531, 1996.

[2] S. S. Sapatnekar, “A Timing Model Incorporating the Effect of Crosstalk on Delay and its Application to Optimal Channel Routing”, in *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, pages 550-559, 2000.

[3] R. Arunachalam, K. Rajagopal, and L. T. Pilleggi, “Taco: Timing analysis with coupling”, in *IEEE/ACM Design Automation Conference*, pages 266-269, 2000.

[4] H. Zhou, N. Shenoy and W. Nicholls, “Timing Analysis with Crosstalk is a Fixpoint on a Complete Lattice”, in *IEEE/ACM Design Automation Conference*, pages 714-719, 2001.

[5] P.D. Gross, R. Arunachalam, K. Rajagopal, L.T. Pileggi, “Determination of worst-case aggressor alignment for delay calculation”, in *IEEE/ACM International Conference on Computer-Aided Design,* pages 212-219, 1998.

[6] L.H. Chen, M. Sadowska, “Aggressor Alignment for Worst-Case Coupling Noise”, in Proceedings of International Symposium on Physical Design, pages 48-54, 2000.

[7] P. Chen, D.A. Kirkpatrick, K. Keutzer, “Miller Factor for Gate-Level Coupling Delay Calculation”, in *IEEE/ACM International Conference on Computer-Aided Design*, pages 68, 2000.

[8] A. B. Kahng, S. Muddu and E. Sarto, “On Switch Factor Based Analysis of Coupled RC Interconnects”, in *IEEE/ACM Design Automation Conference*, pages 79-84, 2000.

[9] J. M. Wang, O. Hafiz, and P. Chen, “A non-iterative model for switching window computation with crosstalk noise”, in *Proceedings of Asia and South Pacific Design Automation Conference*, pages 846-851, 2004.

[10]S. H.Choi, F. Dartu, K. Roy, “Timed Pattern Generation for Noise-on-Delay Calculation”, in *IEEE/ACM Design Automation Conference*, pages 870- 873, 2002.

[11]T. Xiao, M. Sadowska, “Worst Delay Estimation in Crosstalk Aware Static Timing Analysis”, in *IEEE/ACM International Conference on Computer Design*, pages 115-120, 2000.

[12]M.R. Becer, V. Zolotov, R.Panda, A. Grinshpon, I. Algor, R. Levy, C. Oh, “Pessimism reduction in crosstalk noise aware STA”, in *IEEE/ACM International Conferences on Computer Aided Design*, pages 954-961, 2005.

[13]I. Keller, K. Tseng, and N.Verghese, “A Robust Cell-Level Crosstalk Delay Change Analysis,” in *ACM/IEEE International Conferences on Computer Aided Design*, pages 147-154, 2004.

[14]S. Sirichotiyakul, D. Blaauw, C. Oh, R. Levy, V. Zolotov, J. Zuo, “Driver modeling and alignment for worst-case delay-noise”, in *ACM/IEEE Design Automation Conference*, pages 720-725, 2001.

[15]J.F. Croix and D.F. Wong, “Blade and Razor: Cell and interconnect delay analysis”, in *ACM/IEEE Design Automation Conference,* pages 386-389, June 2003.

[16]R. Levy, D. Blaauw, G. Braca, A. Dasgupta, A. Griashpon, C. Oh, B. Orshay, S. Sirichotiyakul, and V. Zolotov, “ClariNet: A noise analysis tool for deep submicron design”, in *ACM/IEEE Design Automation Conference*, pages 233 - 238, 2000.