Simple Metrics for Slew Rate of RC Circuits Based on Two Circuit Moments

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ABSTRACT

In this paper we introduce simple metrics for the slew rate of an RC circuit based on the first two circuit moments. We develop two new slew metrics, *S2M* (slew with 2 moments) and *scaled S2M*, that provide high accuracy with the advantage of simple closed form expressions. S2M is very accurate for middle and far end nodes but it does not perform as well for near end nodes. Scaled S2M is developed to improve upon S2M for near end nodes and is shown to be highly accurate for near as well as far end nodes. For a large set of nets from an industrial 0.18 μm microprocessor, S2M matches SPICE within 2% on average with 78% of the sinks having less than 1% error. For the same test cases, the average error for scaled S2M is less than 3% with 99.4% of the nodes showing less than 5% error.

Categories and Subject Descriptors

B.7.2 [**Integrated Circuits**]: Design Aids, B.8.2 [**Performance and Reliability**]: Performance Analysis and Design Aids

General Terms

Performance, Design

1. INTRODUCTION

On-chip interconnects are modeled as RC circuits and substantial effort has been expended in the delay prediction of such circuits. Many of the most common delay metrics are based on an analogy between non-negative impulse responses of an RC circuit and the *probability density functions* (PDF). This analogy stems from the fact that the impulse response *h(t)* of an RC circuit (without a resistive path to the ground) satisfies the following conditions [1]:

$$h(t) \geq 0 \qquad \forall t$$

$$\int_0^\infty h(t)dt = 1$$

Since these are sufficient conditions for a function to be a PDF, the impulse response of an RC circuit is a probability density function. Also, since the step response is the integral of the impulse response, the step response can therefore be modeled as a *cumulative density function* (CDF). The median of a PDF is defined such that it corresponds to the 50% point of CDF; hence the 50% delay of an RC circuit under step excitation can be calculated by computing the median of the impulse response. However, it is not trivial to compute the median and hence Elmore proposed using the mean of the impulse response to model the 50% delay [2]. The mean of the impulse response is given by the first moment (described later in Section 2) and is very simple to compute. This has led to the widespread popularity of the Elmore delay metric in performance optimization and delay calculation. However, Elmore delay has been shown to be highly inaccurate in certain cases since it does not consider resistive shielding of downstream capacitance [7].

To enhance delay model accuracy, model order reduction techniques such as asymptotic waveform evaluation (AWE) were developed [4]. Though these techniques provide very high accuracy, their use in the inner-loop during design optimization is limited due to the lack of the closed form expressions. Various two-pole delay metrics based on higher order moments were proposed to compromise between a complex reduced order model and a simple first moment based Elmore approximation [5,6]. An empirical delay metric called D2M was proposed in [7] and was shown to be highly accurate and very efficient.

In [8,9,10], the authors extended the probability interpretation of the impulse response of an RC circuit by fitting it to the PDF of various statistical distributions. Though these approaches show very high accuracy, they are not as efficient as D2M and require pre-constructed look-up tables for delay calculation.

Although there has been a large amount of work focused on accurately modeling 50% delay as described above, the importance of a good slew metric has been overlooked. Modeling transition time degradation in RC circuits is an important part of modeling propagation delay. In static timing analysis, slew must be propagated to the next stages because gate and interconnect delays are sensitive to the input slews. Coupling noise and its impact on timing also strongly depends on the transition times of aggressor nets. Accurate crosstalk noise modeling therefore requires precise knowledge of the aggressor slews at the location of the coupling. As noted in [11], the only slew metric currently available in the literature is the Elmore based slew metric [12]. This metric is derived from a single dominant pole approximation and is only as accurate as the first order delay metrics. Elmore [2] and subsequently Gupta [3] noted that the transition time of an RC circuit is similar to the standard deviation of the impulse response but no formal treatment or model investigation has been performed. In this paper, we propose a new two-moment based slew metric called S2M (slew with 2 moments), that is highly accurate and efficient enough for use in physical design optimization loops. We also propose a modified version of this metric called scaled S2M that is equally efficient and provides much better accuracy at near end nodes. Since some existing delay metrics can be extended to compute slews, we perform a thorough investigation of the accuracy of various slew metrics for the first time. The results imply that S2M and scaled S2M exhibit higher accuracy than Elmore and D2M and are typically more accurate than the more complex Weibull metric [10].

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2. BACKGROUND

The new S2M metric is derived using a probability interpretation of the impulse response, hence we begin by reviewing some concepts of circuit moments and probability moments.

Let *h(t)* be the impulse response of an RC circuit and *H(s)* be the Laplace transform of *h(t)*. From the definition of the circuit moments, we have

$$H(s) = m_0 + m_1 s + m_2 s^2 + m_3 s^3 + ... = \sum_{k=0}^{\infty} m_k s^k \qquad (1)$$

Also, since *H(s)* is the Laplace transform of *h(t)*

$$H(s) = \int_0^{\infty} h(t) e^{-st} dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} s^k \int_0^{\infty} t^k h(t) dt \qquad (2)$$

The *k*-th probability moment $\hat{m}_k$ is defined as

$$\hat{m}_k = \int_0^{\infty} t^k h(t) dt \qquad (3)$$

By comparing (1) and (2) and using the above definition of the probability moments, the relationship between the circuit moments m_k and the probability moments $\hat{m}_k$ can be expressed as

$$m_k = \frac{(-1)^k}{k!} \hat{m}_k \qquad (4)$$

The circuit moments of an RLC tree can be computed efficiently by path tracing. The *p*-th order circuit moment (*p*>1) of a node *i* (m_p^i) in an RLC tree can be expressed as

$$m_p^i = \sum_k \left(- R_{ik} C_k m_{p-1}^i - L_{ik} C_k m_{p-2}^i \right) \qquad (5)$$

Here, summation is taken over all the nodes other than the source node. C_k is the capacitance at node *k* and R_{ik} (L_{ik}) denote the total overlap resistance (inductance) in the unique paths from the source node to the nodes *i* and *k*.

The circuit moments can be computed recursively using (5). Since circuit moments are related to the probability moments, hence important information about the RC impulse response PDF can be obtained from these circuit moments. The first probability moment, which is the negative of the first circuit moment, represents the *mean (μ)* of a PDF. Higher order probability moments of the distribution are usually translated into *central moments*. The central moments are the moments around the mean and they contain important geometrical information about the probability density function. The *k*-th central moment of a PDF *h(t)* with the mean μ can be expressed as

$$\mu_k = \int_0^{\infty} (t - \mu)^k h(t) dt \qquad (6)$$

Using above definition of the central moments, the first few central moments can be expressed in terms of circuit moments.

$$\begin{aligned} \mu_2 &= 2m_2 - m_1^2 \\ \mu_3 &= -6m_3 + 6m_1 m_2 - 2m_1^3 \end{aligned} \qquad (7)$$

μ_2 represents the *variance* of the distribution. It is a measure of the spread or the dispersion of the curve from the center. μ_3 is the measure of the *skewness* of the distribution.

3. THE S2M SLEW METRIC

3.1 Metric Derivation

The central moments of a PDF define its shape characteristics. Focusing on the slew rate, variance is the most important central moment as it represents the spread around the mean. This point is the basis of the new S2M metric.

Since the step response of an RC circuit is a cumulative density function, it can be modeled by any monotonic function *F(t)* that satisfies the following conditions:

$$\begin{aligned} & 0 \le F(t) \le 1 \\ & \lim_{t \to -\infty} F(t) = 0 \\ & \lim_{t \to \infty} F(t) = 1 \end{aligned} \qquad (8)$$

RC responses resemble an exponential waveform; hence we model the step response CDF as

$$F(t) = 1 - e^{\frac{-t}{\tau_r}} \qquad (9)$$

The PDF of the impulse response *h(t)* corresponding to this CDF (step response) can be obtained as

$$h(t) = \frac{d}{dt} F(t) = \frac{1}{\tau_r} e^{\frac{-t}{\tau_r}} \qquad (10)$$

The mean μ and the variance μ_2 ($=\sigma^2$) of this impulse response PDF can be easily computed.

$$\begin{aligned} \mu &= \int_0^{\infty} \frac{t}{\tau_r} e^{\frac{-t}{\tau_r}} dt = \tau_r \\ \mu_2 &= \int_0^{\infty} (t - \mu)^2 \frac{e^{\frac{-t}{\tau_r}}}{\tau_r} dt = \tau_r^2 \end{aligned} \qquad (11)$$

We have one unknown parameter τ_r and we obtain its value by matching the variance in (11) with the variance of actual RC circuit impulse response (7). This is the key step in our approach because variance is a measure of slew and it must be preserved for accurate slew analysis.

$$\tau_r = \sqrt{2m_2 - m_1^2} \qquad (12)$$

This value of τ_r can be substituted in (9) and the step response function *F(t)* can be solved for 10 and 90% points. The resulting slew metric is given by

$$S2M = (\ln 9)(\sqrt{2m_2 - m_1^2}) \qquad (13)$$

Regarding the stability of S2M, it can be seen from the definition of the second central moment (variance) in (6) that it is positive for any PDF. Hence the square root term in (12) and (13) is always defined and the metric is stable.

We point out here that if we obtain τ_r by matching the first moment (mean), we obtain a dominant pole approximation where the dominant pole is the inverse of the Elmore term. The scaled Elmore delay (ln 2 * Elmore) and the Elmore slew metric (ln 9 * Elmore) are based on this dominant pole approximation. It should also be noted that we do not propose *F(t)* as an output waveform model and solving *F(t)* for 50% delay may not provide accurate results. Our goal is to find a closed form slew metric that is more accurate than scaled Elmore but retains most of its simplicity. For delay calculations, any two moment-based delay metric such as D2M can be used. In fact, using S2M in conjunction with D2M

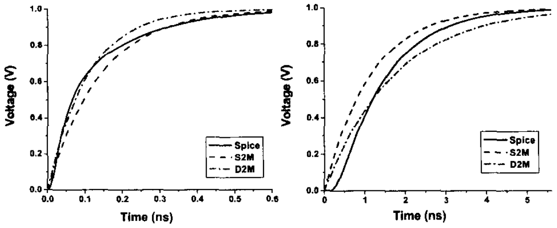

Figure 1. RC circuit response waveforms and their S2M and D2M approximations.

provides a very efficient way to compute delays and slews without complex modeling of the output waveform. In static timing analysis, we are interested only in 50% delay and 10-90% slew rather than the waveform shape. Modeling approaches such as *h*-gamma and AWE attempt to capture the entire output waveform shape but when these waveforms are propagated to the next stages, they are approximated as saturated ramps; hence the extra computational cost of such approaches cannot always be justified.

It may also seem that approximating step response with one pole can cause high error in slew estimation because the actual response can deviate significantly from a single pole exponential approximation. Though it is correct that actual waveforms may deviate from a one pole exponential, their relative spread around the mean and hence the slew is same because their variances were preserved during one pole approximation. Figure 1 shows two RC response waveforms and their S2M and D2M approximations. It is clear from the plots that in these cases, waveforms deviate significantly from a one-pole exponential; however, in both the cases the 10-90% slew is modeled accurately by S2M and 50% delay is modeled well by D2M. Though neither S2M nor D2M match the complete output waveform shape, together they can capture the key delay points needed for timing analysis.

It should be noted here that S2M metric is effectively a scaled form of the standard deviation of the impulse response. The potential relationship between slew rate and the standard deviation of *h(t)* has been suggested previously in [2,3]. Our S2M metric is just a simple extension of this relationship.

3.2 The Scaled S2M Metric

S2M works very well for middle and far end nodes (as will be shown later), however it is not as accurate at near end nodes. In this section, we propose a modified version of S2M called *scaled S2M,* that works extremely well for near as well as far end nodes, while maintaining the efficiency of S2M.

Scaled S2M is derived using an analogy between S2M as a slew metric and scaled Elmore (ln 2 * Elmore) as a delay metric. Both these metrics are single pole approximations of an RC response and can be derived by matching the second and first central moments of the RC impulse response respectively. However, S2M is much more accurate in slew prediction than scaled Elmore is in delay prediction. This is because S2M uses two circuit moments while scaled Elmore is a simpler single moment based metric.

Another similarity in these metrics is that scaled Elmore uses the mean to approximate 50% delay (median) while S2M uses the standard deviation to approximate 10-90% slew. These approximations are accurate for middle and far end nodes but they do not hold as well for near end nodes as near end step responses frequently show long tails due to resistive shielding effects. Step responses with long tails translate to impulse responses with long tails and large skewness (third central moment). Because such impulse responses are highly asymmetric, using the mean and standard deviation to approximate the 50% delay and 10-90% slew becomes inaccurate. Also, near end impulse response PDFs are highly skewed in the positive direction causing S2M and scaled Elmore to significantly overestimate slew and delay at these nodes.

The authors in [7] observed the trend that scaled Elmore significantly overestimates delay at the near end while slightly underestimating delay at the far end. Hence they proposed the D2M metric by introducing an empirical multiplicative factor $r = -m_1 / \sqrt{m_2}$ to the scaled Elmore metric. This factor was found to be much less than one for near end nodes and slightly greater than one for far end nodes. Using this factor, the accuracy in delay prediction is vastly improved. S2M also shows a similar trend of overestimation at near end although it is extremely accurate at far end nodes. Based on these observations relating scaled Elmore as a delay metric and S2M as a slew metric, we conclude that a similar factor can be used to improve near end accuracy. This is the basis of a second new slew metric called *scaled S2M.*

Empirically we have found a good multiplicative factor to be $\sqrt{-m_1 / \sqrt{m_2}}$, which is the square root of the factor proposed to improve scaled Elmore [7]. This is intuitive since S2M is a variance-based metric while scaled Elmore is mean-based. Also S2M shows better accuracy than scaled Elmore and hence the multiplicative factor for S2M should be less than that for scaled Elmore (i.e., closer to 1). The new scaled S2M metric is given by

$$Scaled\ S2M = \frac{\sqrt{-m_1}}{\sqrt[4]{m_2}}(\ln 9)(\sqrt{2m_2 - m_1^2}) \tag{14}$$

Figure 2 shows a simulated near end RC response and both S2M and scaled S2M approximations for a difficult near end case. It is clear that S2M overestimates slew due to the long tail. Scaled S2M shows much better 10-90% fit for this test case.

The S2M and scaled S2M metrics can be easily extended to ramp inputs by using the PERI (*P*robability distribution function *E*xtension for *R*amp *I*nputs) approach described in [11]. The PERI technique is based on the fact that when two mutually independent PDFs are convolved, their mean and central moments add. By using this simple observation, the authors of [11] propose the following expression to extend step slew metrics to ramp inputs.

$$Slew(Ramp) = \sqrt{Slew^2(step) + T_R^2} \tag{15}$$

Here, T_R is the input slew and *Slew(step)* and *Slew(ramp)* denote slew for step and ramp excitation respectively.

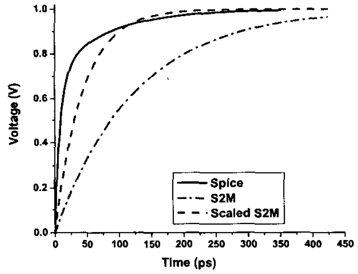

Figure 2. RC circuit response waveform compared to S2M and scaled S2M approximations.

4. EXPERIMENTAL RESULTS

In this section, we compare our results with the Elmore based slew metric (ln 9 * Elmore) and also with other delay metrics that can be extended to compute slew. Among major delay metrics, only D2M and Weibull are extendable to slew. Though *h*-gamma can also be extended to compute slews it requires extra look-up tables for 10% and 90% points and the generation of these tables is non-trivial. Hence, we do not compare our results with *h*-gamma.

A. Simple RC Tree Test Case

First we consider a simple RC tree taken from [7,10] and shown in Figure 3. Table I shows that S2M results match well with SPICE, with very high accuracy at far end nodes but relatively larger errors at near end. Scaled S2M works well for all nodes including very small errors at the near end. Weibull also shows good match with SPICE for this test case although it underestimates the near-end node slew rate by over 10%.

B. Results on Industrial Nets

Next we consider a large number of RC interconnect examples from a 0.18 μm high performance microprocessor design. The total number of sinks considered was 3702. Table II shows the mean, standard deviation, and error bins for different metrics (errors are taken as absolute values when computing the mean). The average error of S2M is much better than all other metrics. Scaled S2M also shows low average error with the smallest standard deviation. Since most of the nodes tested are far end sinks, scaled S2M and S2M show comparable results. Table II also shows that S2M predicts the slew rate of 78% of the sinks within 1% of SPICE whereas the other metrics only do so for 1-12% of cases. Scaled S2M is also highly accurate with 99.4% of the nets showing less than 5% error.

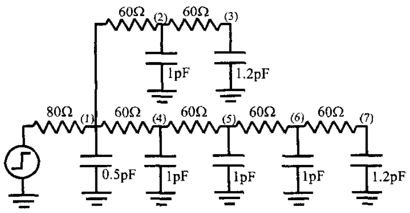

Figure 3. A simple RC tree.

Table I. Comparison between different slew metrics for test circuit of Figure 3. Slew rates are in ps and error relative to SPICE is given in parentheses.

Node	Elmore	D2M	Weibull	Scaled S2M	S2M	Spice
1	1212 (–24%)	947 (–40.6%)	1426 (–10.5%)	1616 (1.3%)	1828 (14.7%)	1594
2	1502 (–12%)	1333 (–21.9%)	1663 (–2.6%)	1655 (–3%)	1865 (9.3%)	1707
3	1661 (–3.7%)	1559 (–9.6%)	1762 (2.1%)	1757 (1.8%)	1872 (8.5%)	1725
4	1766 (–10.8%)	1628 (–17.8%)	1902 (–3.9%)	1973 (–0.3%)	2055 (3.8%)	1980
5	2188 (1.4%)	2207 (2.3%)	2164 (0.3%)	2159 (0%)	2149 (–0.4%)	2158
6	2478 (11.8%)	2631 (18.7%)	2288 (3.2%)	2247 (1.3%)	2180 (–1.6%)	2216
7	2636 (18%)	2870 (28.5%)	2340 (4.8%)	2281 (2.1%)	2186 (–2.1%)	2233

Table II. Error statistics and Error bins of slew metrics on industrial nets (at 3702 sinks).

Error Statistics					
	Elmore	**D2M**	**Weibull**	**Scaled S2M**	**S2M**
Mean	16.1%	24.9%	5.3%	2.97%	1.3%
Std. Deviation	11.3%	18.1%	3.7%	1.38%	3.1%
Error Bins					
	Elmore	**D2M**	**Weibull**	**Scaled S2M**	**S2M**
< 1%	1.9%	0.9%	6%	11.3%	78%
< 5%	9%	6.5%	37.2%	99.4%	93.5%
< 10%	20.4%	11.6%	99.1%	99.9%	97.1%

5. CONCLUSIONS

This paper describes two new metrics to accurately compute the slew rate at any node in an RC circuit. The metrics, named S2M and scaled S2M, are simple closed-form functions of the first two circuit moments. These metrics should be useful in a range of physical design areas such as noise avoidance during routing, on-the-fly wire resizing/spacing, and fast checking of slew rate constraints.

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