Statistical Modeling of Cross-Coupling Effects in VLSI Interconnects

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Abstract— In this paper, we develop an approach for statistical modeling of crosstalk noise and dynamic delay degradation in coupled RC interconnects under process variations. The proposed model enables closed-form computation of mean and variance of noise peak and worst case dynamic delay for given variabilities in physical dimensions. We compare the proposed model against HSPICE Monte Carlo simulations and report an average error in mean and standard deviation of noise peak to be 2.7% and 3.7% respectively.

1. INTRODUCTION

The importance of process variation has been growing rapidly with the scaling of process dimensions [1]. Lately, there has been a lot of work in modeling impact of process variations on performance and power. However, one area that has not been investigated in detail is the impact of process variation on reliability issues such as crosstalk noise, crosstalk delay, electromigration and IR drop. Process variations can affect these parameters significantly causing unexpected reliability failures in the manufactured products.

In this work, we focus on modeling the impact of process variation on crosstalk noise glitch and delay change due to simultaneous switching (dynamic delay). It is not trivial to apply traditional corner based analysis to these problems because it is hard to identify a worst case combination of process parameters that results in the worst case coupling noise. Even if we could identify a worst case combination, choosing a 3-sigma point for all parameters usually results in an overly pessimistic design. In this paper, we develop a probabilistic model to account for interconnect process variations in crosstalk noise analysis. The proposed model allows us to express means and variances of noise peak and dynamic delay in a closed-form manner and hence can be very useful in statistical noise related physical design optimizations.

Accurate statistical modeling of interconnect coupling requires accurate nominal models for interconnect delay, crosstalk noise and dynamic delay. Modeling interconnect delay has been a much studied topic and various highly accurate models exist in literature [3,4,5,6]. Similarly, various static noise models have also been proposed that exhibit a high degree of accuracy [7,8]. On the other hand, accurate estimation of dynamic delay still remains a challenging task. In this work, we develop a new nominal closed-form dynamic delay model that uses advanced waveform models like Weibull along with worst-case alignment to obtain peak crosstalk induced delay degradation. This new model along with existing static noise and delay models is then used to obtain statistical models of coupling effects under interconnect variations.

The statistical modeling approach used in this work is similar to the variational delay metric proposed in Reference [2]. We also express noise and dynamic delay directly as a function of changes in the physical parameters. This formulation preserves all correlations and can be very useful in evaluating noise sensitivities due to changes in various physical dimensions. Our approach is based on the observation that if the variations in different physical dimensions (wire width, wire thickness and interlayer dielectric thickness etc.) are assumed to be independent normal random variables, then the coupling noise and the dynamic delay also tend to have Gaussian distribution. This allows us to express these distributions as a linear function of variations in physical dimensions.

2. BACKGROUND

The first step in any statistical interconnect modeling approach is to capture the effect of geometric variations on the electrical parameters (resistances and capacitances) of the wires. It has been shown that this effect can be accurately captured by the simple linear approximation shown in Equation 1 [2].

$$R = R_{nom} + \left(\frac{\partial R}{\partial W}\right)_{nom} \Delta W + \left(\frac{\partial R}{\partial T}\right)_{nom} \Delta T$$
$$C = C_{nom} + \left(\frac{\partial C}{\partial W}\right)_{nom} \Delta W + \left(\frac{\partial C}{\partial T}\right)_{nom} \Delta T + \left(\frac{\partial C}{\partial H}\right)_{nom} \Delta H \qquad (1)$$

Here, R_{nom} and C_{nom} represent nominal resistance and capacitance values. ΔW, ΔT, and ΔH represent the change in metal width, metal thickness, and ILD thickness respectively. For simplicity, throughout the discussion in this work, we consider variations only in metal width (W), metal thickness (T) and interlayer dielectric thickness (H) but there is no restriction on the number of variables in our methodology and our approach can be easily extended to include other variation sources. The metal-to metal spacing is considered to be inversely correlated with variation in metal width. We also assume that variations in various physical dimensions have Gaussian distributions and variability in one physical dimension is mutually independent with variations in other dimensions.

Reference [2] shows that under these assumptions, circuit moments can also be expressed as linear functions of process variations.

$$m_1 = m_{1(nom)} + k_w \Delta W + k_T \Delta T + K_H \Delta H$$
$$m_2 = m_{2(nom)} + A_w \Delta W + A_T \Delta T + A_H \Delta H \qquad (2)$$

Here, m_1 and m_2 represent first and second circuit moments under process variations. The linear coefficients of the circuit moments in Equation 2 can be easily computed using methodology discussed in [2]. We use these simple linearized models of circuit moments in this work. With this background, now we propose statistical noise model in the following section.

3. STATISTICAL MODELING OF STATIC NOISE

Static noise is defined as the noise pulse induced on a quite victim net due to switching of neighboring aggressors. To first order, the magnitude of static noise is directly proportional to the ratio of coupling capacitance to ground capacitance. This causes noise magnitude to be very sensitive to variations in metal width and inter-wire spacing and small variations in these dimensions can result in large fluctuations in the noise peak. To demonstrate this claim, we consider a simple coupled RC interconnect testcase. The width (W), thickness (T), spacing (S), and interlayer dielectric layer thickness (H) for the interconnect lines were randomly chosen to be 0.4μ, 0.75μ, 0.45μ and 0.3μ respectively. Moreover, the variation in W, T and H were taken to be 25%, 21%, 17% of the nominal values respectively (recall that we consider W and S to be perfectly inversely correlated). Figure 1 shows the spread of noise waveforms obtained using HSPICE Monte Carlo simulations. The figure shows that for this testcase, the noise peak varies from approximately 160 mV to 235 mV, thereby implying that variation in noise due to process variation can be significant and should be modeled accurately.

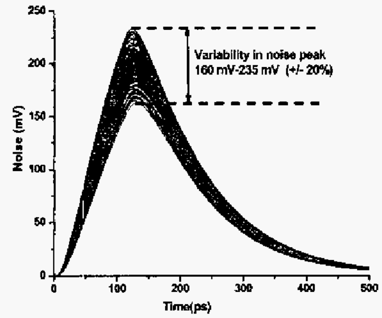

Fig 1: Crosstalk noise waveforms as obtained from Monte Carlo simulations.

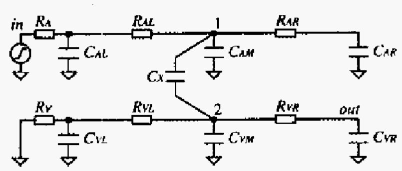

Fig 2: 6-node circuit template for coupling noise estimation as used in Reference [7]

We use the nominal static noise peak model proposed in [7] for our analysis. This model approximates victim and aggressor RC distributed interconnects by a 6-node symmetric template as shown in Figure 2. By solving this template, the noise peak induced on the victim line is given by

$$Noise_{peak} = \frac{t_X}{t_R} \cdot \frac{\left(1 - e^{-t_R/t_V}\right)^{\beta}}{\left(1 - e^{-t_R/t_A}\right)^{\alpha}} \tag{3}$$

Where

$$t_X = C_X \cdot (R_V + R_{VL}); \quad \alpha = \frac{t_V}{t_V - t_A}; \quad \beta = \frac{t_A}{t_V - t_A}$$

$$t_V = C_{VL} \cdot R_V + (C_{VM} + C_X) \cdot (R_V + R_{VL}) + C_{VR} \cdot (R_V + R_{VL} + R_{VR})$$

$$t_A = C_{AL} \cdot R_A + (C_{AM} + C_{veff} + C_{reff}) \cdot (R_A + R_{AL})$$

Further, t_R is the rise time for the saturated ramp input signal at the aggressor line and C_{reff} and C_{veff} are the effective capacitances for the right half of the aggressor and the victim line respectively as discussed in Reference [7].

In the presence of process variations, all resistances and capacitances shown in Figure 2 become correlated random variables. These random variables are modeled as linear functions of changes in physical dimensions as discussed in Equation 1. These resistances and capacitances can then be substituted in t_X, t_V and t_A expressions. These expressions are then simplified by neglecting higher-order terms of $\Delta W, \Delta T$ and ΔH. This assumption is highly accurate because $\Delta W, \Delta T$ and ΔH are small quantities and hence higher-order terms containing multiplications of these quantities can be safely ignored. Under these assumptions, t_X, t_V and t_A are given by

$$\begin{aligned} t_X &= (T_X + \Delta T_X) = T_X + k_w \Delta W + k_T \Delta T + k_H \Delta H \\ t_V &= (T_V + \Delta T_V) = T_V + v_w \Delta W + v_T \Delta T + v_H \Delta H \\ t_A &= (T_A + \Delta T_A) = T_A + a_w \Delta W + a_T \Delta T + a_H \Delta H \end{aligned} \tag{4}$$

Here, T_X, T_V, T_A are nominal quantities. The results from Equation 4 can now be substituted in the $Noise_{peak}$ expression of Equation 3. This new expression is stepwise reduced by using Taylor series expansion. At each reduction step, higher order product terms containing $\Delta W, \Delta T$ and ΔH are ignored. Finally, $Noise_{peak}$ is expressed as a linear function of $\Delta W, \Delta T$ and ΔH.

$$Noise_{peak} = (Noise_{peak,nom} + \Delta Noise_{peak}) \tag{5}$$

$$\Delta Noise_{peak} = Noise_{peak,nom} \cdot \left(\frac{\Delta T_X}{T_X} + M \Delta T_V + N \Delta T_A \right)$$

$$M = \frac{B}{T_V - T_A} \cdot \ln\left(\frac{1 - e^{-T_R/T_V}}{1 - e^{-T_R/T_A}} \right) + \frac{A \cdot T_R}{T_A^2} \cdot \frac{e^{-T_R/T_A}}{\left(1 - e^{-T_R/T_A}\right)} \qquad N = \frac{-A}{T_V - T_A} \cdot \ln\left(\frac{1 - e^{-T_R/T_V}}{1 - e^{-T_R/T_A}} \right) - \frac{B \cdot T_R}{T_V^2} \cdot \frac{e^{-T_R/T_V}}{\left(1 - e^{-T_R/T_V}\right)}$$

$$A = T_A / (T_V - T_A) \qquad B = T_V / (T_V - T_A)$$

The above formulation is linear with respect to variations in physical dimensions, thereby implying that for normal distribution functions for W, T and H,, the distribution of $Noise_{peak}$ is also normal. Although, this result is obtained by making a number of approximations in performing the long reduction, we will show that this normal distribution result matches well with SPICE Monte Carlo simulations (shown in Section 5).

4. DYNAMIC DELAY MODELING

Dynamic delay is defined as signal delay at the victim output when both aggressor and victim switch simultaneously. The aggressor switching can either slow down or speed up the victim depending on its switching polarity with respect to the victim. The victim gets slower in case of out of phase switching and faster during in-phase switching. The delay push-out during out of phase switching can cause timing failures and hence must be accounted for in order to ensure correct operation of the circuit.

4.1 Nominal Dynamic Delay Model

In this section, we develop a new dynamic delay model that uses accurate waveform modeling along with worst-case alignment to compute worst case delay noise. Our dynamic delay model is based on the superposition principle as in [10,14]. Hence, accurate estimation of dynamic delay requires accurate modeling of static noise and isolated victim switching waveform. Static noise can be modeled using approach discussed in the previous section. However, modeling of victim switching waveform is a more complex problem. A Weibull function based waveform model was proposed in [13]. The Weibull function is given as:

$$y = 1 - e^{-\left(\frac{t}{\beta}\right)^{\alpha}} \tag{6}$$

In order to fully characterize Weibull function, we match delay and slew of actual waveform with the above expression. If we denote the 50% Vdd crossing time of the output waveform as D and 10% - 90% slew as S. Then we have:

$$D = \beta \cdot (\ln 2)^{\frac{1}{\alpha}} \qquad S = \beta \cdot \left((\ln 10)^{\frac{1}{\alpha}} - \left(\ln \frac{10}{9} \right)^{\frac{1}{\alpha}} \right) \tag{7}$$

The values for D and S can be computed analytically using any of the existing delay and slew metrics [3,4,5,6]. Now, these values of D and S can be used in Equation 7 to solve for α and β. However, the problem is that the Equation set 7 can only be solved by using numerical techniques. We make following simplifications to solve Equation 7 in closed form manner.

By dividing S by D we get:

$$\frac{S}{D} = \frac{\left((\ln 10)^{\frac{1}{\alpha}} - \left(\ln \frac{10}{9} \right)^{\frac{1}{\alpha}} \right)}{(\ln 2)^{\frac{1}{\alpha}}} \tag{8}$$

The above expression of S/D is evaluated for practical range of α (1.4-2.4) as obtained through simulations over a large set of testcases. S/D can now be fitted by a simple 2nd order polynomial.

$$\frac{S}{D} = 0.5392\alpha^2 - 2.9274\alpha + 5.124 \tag{9}$$

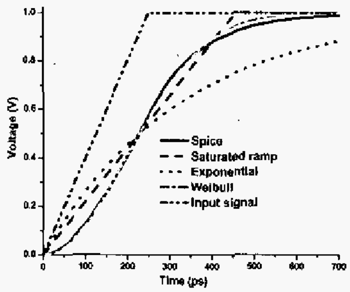

Fig 3: Output waveform of an RC circuit under ramp input and various analytical approximations of the waveform.

Equation 9 can be easily solved to compute α in terms of *S/D*. The value of α thus obtained is then substituted in Equation 7 to estimate β. The values of α and β computed in this manner can now be used in Equation 6 to fully characterize the Weibull waveform model of an isolated victim.

To verify this theory, we compare the analytical result with HSPICE simulations. Figure 3 shows that the Weibull waveform obtained using above method fits actual waveform very accurately. The figure also shows that simple ramp or exponential expressions are highly inaccurate and hence using them for dynamic delay prediction can result in highly erroneous results.

Once the waveforms are modeled accurately, the next step in dynamic delay modeling is to superimpose the static noise on the isolated victim delay waveform. This superposition should be done in a manner that results in maximum delay shift to model the worst-case switching behavior. This is achieved by aligning the aggressor and the victim switching such that the time when the static noise reaches its maximum value ($Noise_{peak}$) is matched to the time when the isolated victim delay waveform crosses the ($0.5V_{DD}$ + $Noise_{peak}$) value [12]. For the Weibull model, the above condition results in the following equation for 50% switching time (t_{50}) of the victim waveform.

$$1 - e^{-\left(\frac{t_{50}}{\beta}\right)^{\alpha}} = \frac{1}{2} + Noise_{peak} \tag{10}$$

The above expression of t_{50} can now be used to solve for worst case dynamic delay (x_{delay})

$$x_{delay} = \beta \ln^{\frac{1}{\alpha}}\left(\frac{1}{0.5 - Noise_{peak}}\right) - \frac{T_R}{2} \tag{11}$$

The x_{delay} thus computed gives the final expression for estimating the worst-case dynamic delay due to simultaneous switching of aggressor and victim wires. The overall modeling flow is summarized below.

For a given coupled RC network, perform the following steps to compute worst-case dynamic delay:

- Compute slew (*S*) and delay (*D*) of an isolated victim line.
- Model the victim waveform by the Weibull model. Compute Weibull parameters α and β using Equations 7 and 9.
- Compute noise peak $Noise_{peak}$ using Equation 3.
- Use worst-case victim and aggressor alignment to calculate worst-case dynamic delay using Equation 11.

4.2 Statistical Dynamic Delay Model

In the previous section, we discussed a methodology to estimate worst-case dynamic delay in coupled RC interconnects. For statistical modeling of delay noise, we begin with this nominal dynamic delay modeling methodology. Based on the above modeling flow, the first step is to compute slew (*S*) and delay (*D*) under process variations. These can be easily computed by using approach discussed in Reference [2]. The next step is to compute statistical expressions for α, β and $Noise_{peak}$. Under a linear assumption, α, β, and $Noise_{peak}$ can be replaced with

$$\begin{aligned} \alpha &= \alpha_{nom} + \Delta\alpha \\ \beta &= \beta_{nom} + \Delta\beta \\ Noise_{peak} &= Noise_{peak,nom} + \Delta Noise_{peak} \end{aligned} \tag{12}$$

Expressions for $\Delta Noise_{peak}$ have already been given in Equation 5. $\Delta\alpha$, $\Delta\beta$ are formulated by substituting *S* and *D* in Equations 7 and 9. After truncating to retain only linear terms, the final reduced formulation for α and β is given by

$$\alpha = \alpha_{nom} - \frac{2.1568 \times \left(\Delta S - \frac{S}{D} \cdot \Delta D\right)}{(4 \times 0.5392) \cdot D \cdot \sqrt{(2.9274)^2 - 4 \times 0.5392 \times \left(5.124 - \frac{S}{D}\right)}} \tag{13}$$

$$\beta = \beta_{nom} + \frac{\left(\Delta D + \frac{D}{\alpha^2} \ln(\ln 2) \cdot \Delta\alpha\right)}{(\ln 2)^{\frac{1}{\alpha}}}$$

Here, ΔD and ΔS represent variabilities in delay and slew.

Now, we can substitute α, β and $Noise_{peak}$ in Equation 11. Once again, under the Gaussian (linear) assumption the final expression for dynamic delay can be expressed as

$$x_{delay} = \left(\frac{1}{0.5 - N_{peak}}\right) \times \left(1 + \frac{\Delta\beta}{\beta} - \frac{\Delta N_{peak}}{\alpha \cdot (0.5 - N_{peak}) \cdot \ln(0.5 - N_{peak})} - \frac{\Delta\alpha}{\alpha^2} \ln(-\ln(0.5 - N_{peak}))\right) - \frac{T_R}{2} \tag{14}$$

Equations 11 and 14 are our final results for nominal and statistical delay noise modeling. In the next section, these results are verified against HSPICE simulations.

5. RESULTS

In this section, we compare the static noise and dynamic delay models developed in Sections 3 and 4 against HSPICE simulations. All simulations were performed using 1V, 130nm technology.

First, we verify the statistical noise peak model proposed in Equation 5 against HSPICE Monte-Carlo simulations. As an example, we consider a coupled RC interconnect testcase and apply a saturated ramp input at the aggressor line with a rise time of 110ps. The width, thickness, spacing, and interlayer dielectric thickness are randomly taken to be 0.55μ, 0.44μ, 0.4μ, and 0.3μ respectively. The 3-σ normal variabilities in the above mentioned parameters are also randomly chosen as 14%, 30% and 15% of the nominal values, respectively. Again, spacing is considered to be in inverse relation to width (pitch is constant). Our model is based on the assumption that under normal distributions of process variations, the static peak noise distribution is also Gaussian. To test this assumption, we look at the quantile-quantile (q-q) plot of the noise peak distribution obtained from HSPICE simulations and compare it with the analytical model. Figure 4 shows this comparison and demonstrates that the q-q plots of HSPICE simulations and the analytical model match very well. Figure 5 shows these results using histograms and Gaussian pdfs. Once again it is clear from this figure that noise peak distribution is Gaussian and the mean and variance of this distribution as obtained analytically match well with Monte Carlo results.

Similarly, for testing our dynamic delay model (Equations 11 and 14), we show the probability plots and q-q plots for one randomly selected testcase. These plots are shown in Figures 6 and 7 respectively. The width, thickness, spacing, and interlayer dielectric thickness for this arbitrarily chosen testcase are taken as 0.65μm, 0.5μm, 0.4μm, and 0.26μm respectively, and 3-σ variations for the above parameters were 23%, 11% and 11% of their nominal values respectively. Once again, it is clear from these figures that the Gaussian approximation is accurate and the analytical mean and standard deviation calculated under this assumption match well with SPICE simulations.

Finally, we compare our results for 2300 random testcases generated by varying nominal physical dimensions and their 3-sigma variabilities. For each test case, the 3-sigma variability in various physical dimensions was randomly chosen to be between 10% and 30% of the nominal. Table I shows the average error in mean and standard deviation for these 2300 test cases compared to Monte Carlo simulations. The table shows that the model works extremely well. For these testcases, 3-sigma variation in noise peak ranged from 3% - 40% and the 3-sigma variation in dynamic delay ranged from 3% to 30% of the nominal values. These results imply that variability in coupling noise can be significant and must be modeled accurately.

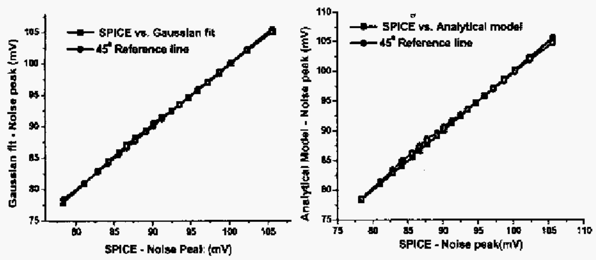

Fig 4: Q-Q plot for noise peak: (Left) SPICE results vs. Gaussian distribution (with same mean and stdev as obtained from SPICE simulations) with 45° reference line. (Right) SPICE result vs. analytical model with 45° reference line.

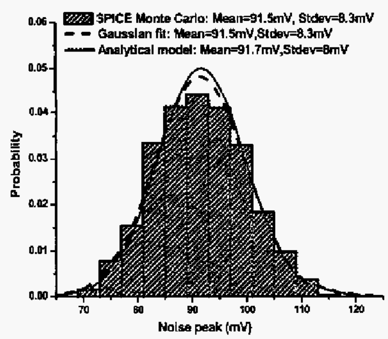

Fig 5: Probability plot of static noise comparing SPICE Monte Carlo simulation results with Gaussian distributions.

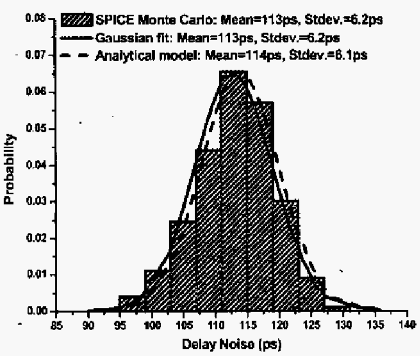

Fig 6: Probability plot of dynamic delay comparing SPICE Monte Carlo simulation results with Gaussian distributions.

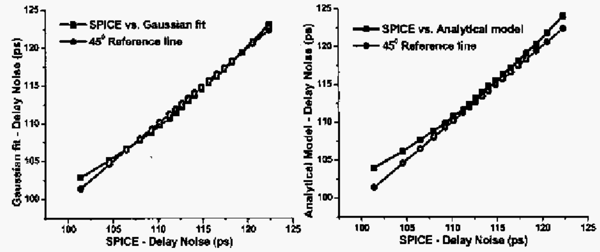

Fig 7: Q-Q plot for dynamic delay: (Left) SPICE results vs. Gaussian distribution (with same mean and stdev as obtained from SPICE simulations) with 45° reference line. (Right) SPICE result vs. analytical model with 45° reference line.

Table I: Error statistics of static noise peak and dynamic delay

2300 testcases	Mean (Noise peak)	Stdev (Noise peak)
Avg. Error.	2.7%	3.7%
	Mean (Dynamic Delay)	Stdev (Dynamic Delay)
Avg. Error	2.6%	12.4%

6. CONCLUSIONS

We proposed an analytical model to estimate mean and variance of coupling noise and dynamic delay in the presence of process variations. The proposed models are based on the assumption that distribution functions of crosstalk noise and delay can be approximated as normal random variables. This allows us to simplify the models by truncating complex expressions to retain only linear terms. We show that this assumption is very accurate and the mean and standard deviation as computed by the proposed model match well with SPICE simulations. Due to its efficiency and accuracy, the proposed analysis can be very useful in statistical noise related physical design optimizations.

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