

## Inductance: Implications and Solutions for High-Speed Digital Circuits

## SE1 Inductance Extraction and Modeling

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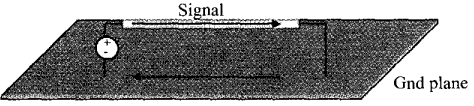
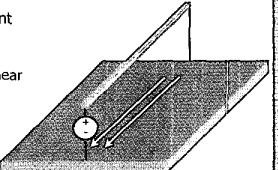
Blaauw

With operating frequencies entering the multi-gigahertz range, inductance is becoming an increasingly important consideration in design and analysis of on-chip interconnects. An overview is given of the extraction and analysis issues related to on-chip inductance effects, starting with a brief overview of Maxwell's equations.

The quasi-static assumption and its implications for analysis of VLSI interconnects are discussed. It is then shown how interconnects can be represented using partial element equivalent circuit (PEEC) models. The complexity of current flow in VLSI circuits is then examined and the PEEC-based model required to represent it is presented. The importance is shown of constructing a comprehensive model that includes substantial portions of the power and ground grid surrounding the signal net to model the distributed current return paths. The importance is shown of modeling the power grid decoupling capacitance, power grid supply pads, signal net coupling capacitance, and local power and ground connections of the driver and receiver gates.

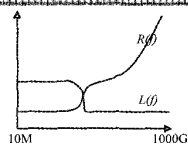
Methods are explained for simulating the constructed model and techniques that can be used to speed-up the simulation of large PEEC models. Simplified approaches are discussed that use so-called loop inductance models, or use RL circuit formulations and are compared with more detailed PEEC models. Results show the trade-off between the accuracy and efficiency of the different methods. The applicability of each method at different stages of the design is discussed. Experimental results are based on simulations of industrial circuits, including a global clock net structure of a large multi-gigahertz processor.

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Outline	Frequency Dependence
<p><b>What is Inductance?</b></p> <ul style="list-style-type: none"> <li>Inductance is a phenomenon created by current traveling in a closed loop</li> <li>Loop inductance is proportional to the area of the loop               <ul style="list-style-type: none"> <li><math>L = \mu A</math></li> </ul> </li> <li>Driver injects current into a wire which returns to the driver through a <i>return path</i> <ul style="list-style-type: none"> <li>power and ground lines (AC ground)</li> <li>substrate</li> <li>other signal lines</li> </ul> </li> </ul> 	<p><b>Frequency Dependence</b></p> <ul style="list-style-type: none"> <li>Currents seek path of least impedance               <ul style="list-style-type: none"> <li><math>Z = R + j\omega L</math></li> </ul> </li> <li>Current return paths are frequency dependent</li> <li>Low frequency:               <ul style="list-style-type: none"> <li><math>R</math> dominates: return current spreads                   <ul style="list-style-type: none"> <li>Low resistance and high inductance</li> </ul> </li> </ul> </li> <li>High frequency:               <ul style="list-style-type: none"> <li><math>j\omega L</math> dominates: return current crowds close</li> </ul> </li> <li>Very high frequency               <ul style="list-style-type: none"> <li>skin effect: current travels near surface of the conductor:                   <ul style="list-style-type: none"> <li><math>R \sim f^{0.5}</math></li> </ul> </li> </ul> </li> </ul> 

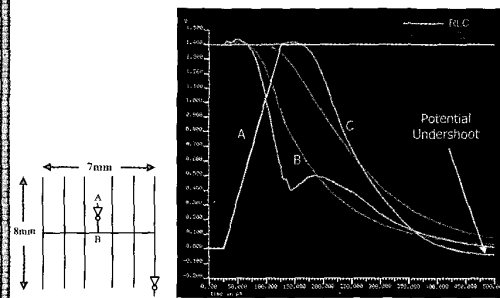
### Frequency Dependence

- As frequency increases:
  - R increases due to proximity and skin effect
  - L reduces due to proximity effect

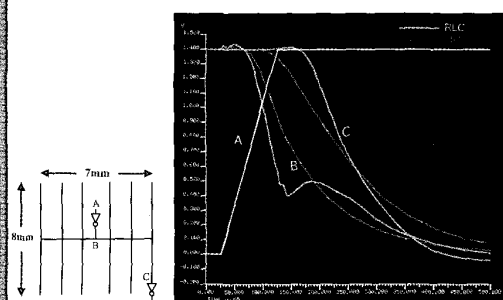


- $Z = R + j\omega L$ 
  - $R \uparrow$
  - $L \downarrow$
  - $\omega \uparrow$
- Signal response increasingly characterized by inductance...

### Inductive Effects



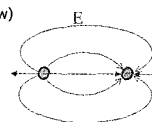
### Inductive Effects



### Electromagnetic Field Equations (Maxwell)

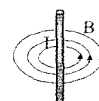
- Relation between electric field & charges (Gauss' law)
  - Electric field starts and ends on charges

$$\nabla \cdot D = \rho$$

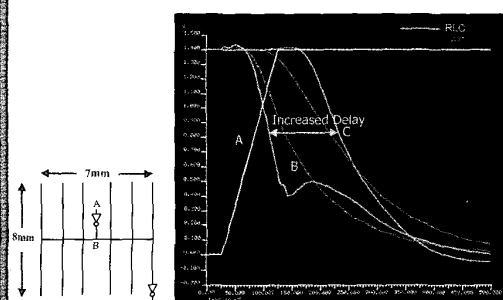


- Source free nature of magnetic field
  - magnetic field forms closed loops

$$\nabla \cdot B = 0$$



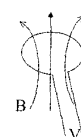
### Inductive Effects



### Maxwell's Equations (con't)

- Electromagnetic induction law (Faraday's law)
  - change in magnetic field induces electric field

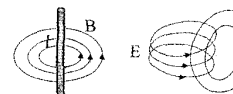
$$\nabla \times E = -\frac{\partial B}{\partial t}$$



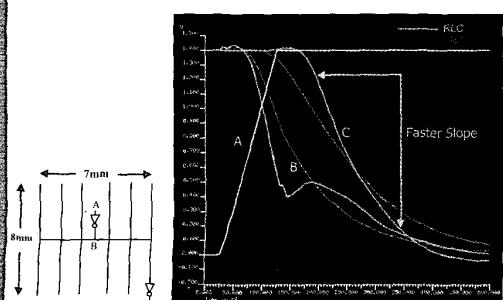
Magnetic field induced by:

- current in conductors (Ampere's law)
- change in electric field (Displacement current)

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$



### Inductive Effects



### Solving Maxwells eqns: full-wave simulation

#### Solutions

- Frequency domain
- Time domain

#### Methods

- Finite Difference Method
- Finite Element Method (FEM)
- Boundary Element Method (BEM)
- Method of moments (MOM)

#### Transmission line models

### Common Simplifying Assumptions

#### Electro-static assumption:

$$\begin{aligned} \nabla D &= \rho \\ \nabla B &= 0 \\ \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times H &= J + \frac{\partial D}{\partial t} \end{aligned}$$

- No current and magnetic field
- Used for capacitance extraction

#### DC assumption:

$$\begin{aligned} \nabla D &= \rho \\ \nabla B &= 0 \\ \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times H &= J + \frac{\partial D}{\partial t} \end{aligned}$$

- Fixed current
- Electric and magnetic field are independent
- Current changes slow: magnetically induced voltage small compared with resistance drop
- Used for RC interconnect simulations

### Quasi-static Assumption

$$\begin{aligned} \nabla D &= \rho \\ \nabla B &= 0 \\ \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times H &= J + \frac{\partial D}{\partial t} \end{aligned}$$

- o Magnetic field change creates significant electric field
- o Assume:
  - Displacement current is small
  - Electric field change does not create significant magnetic field
- o No electromagnetic wave propagation without displacement current
- o Interaction between pairs of elements becomes instantaneous (no retardation)
- o Valid if:
  - displacement current is small compared to current density  $J$  (sufficiently low frequency)
  - characteristic size is smaller than wavelength (10GHz wavelength is 30mm)

### Inductance Definition Using Flux

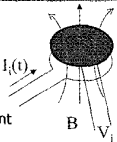
- Traditional definition of inductance in terms of magnetic flux:

$$\Phi_{ij} = \iint_{\text{Current loop area}} B \cdot da$$

- Inductance  $M_{ij}$  is ratio of flux in loop  $j$  over current in loop  $i$

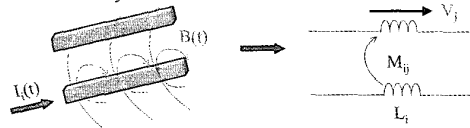
$$\Phi_{ij} = M_{ij} I_i$$

- Flux definition not well suited for on-chip interconnects:
  - system consist of many intersecting loops (no individual loops evident)
  - wire thickness significant compared to loop area (area integral not easily defined)



### Partial Inductance

- Each wire segment is assigned partial self and mutual inductance
  - split wires into segments
  - partial inductance  $M_{ij}$  between wire segments  $i,j$  reflects voltage induced in  $j$  due to current in  $i$

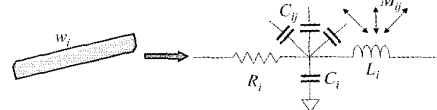


- Analytical solution for partial inductance for parallel filaments with uniform current distribution:

$$M_{ij} = \frac{\mu}{4\pi} \ln \frac{1}{a_i a_j} \int \int \frac{1}{R} dV_i dV_j$$

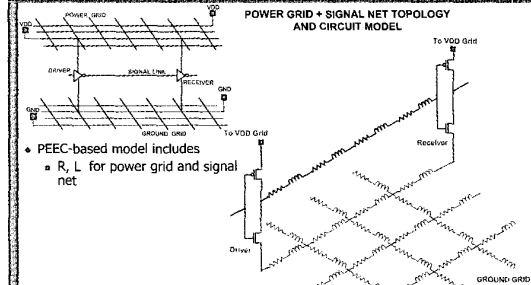
### PEEC Model

- A. Ruehli [IBM Journal of R&D, 1972]
- Discretize all wires for lumped model
- Treat each discrete filament as an equivalent circuit



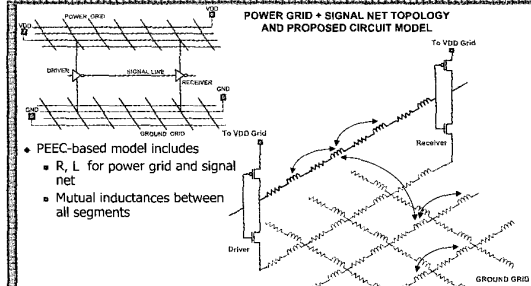
- Compute lumped parameters  $R_i, L_i, M_{ij}, C_i, C_{ij}$

### Full PEEC Based Circuit Model



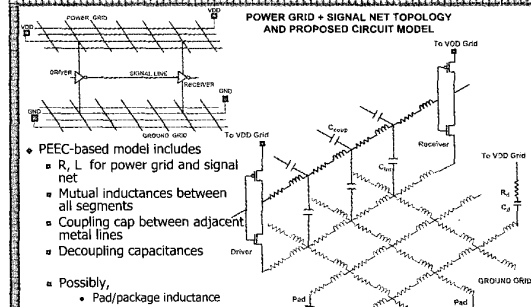
- PEEC-based model includes
  - $R, L$  for power grid and signal net

### Full PEEC Based Circuit Model



- PEEC-based model includes
  - $R, L$  for power grid and signal net
  - Mutual inductances between all segments

### Full PEEC Based Circuit Model

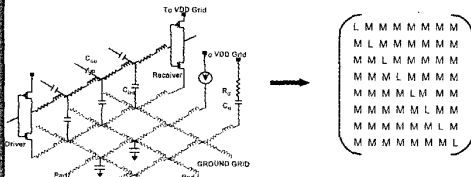


- PEEC-based model includes
  - $R, L$  for power grid and signal net
  - Mutual inductances between all segments
  - Coupling cap between adjacent metal lines
  - Decoupling capacitances
  - Possibly,
    - Pad/package inductance

### PEEC Model Approach - Issues

#### ♦ Fully-dense L matrix

- Computation of all mutual inductances is expensive
- Large simulation time => needs acceleration

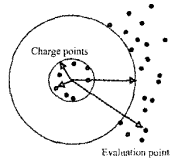


### Sparsification - Implicit

#### ♦ Fast Multipole (FastHenry)

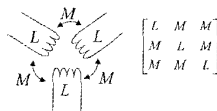
[Kamon IEEE-MTT '94]

- Lump the effect of a bunch of distant conductors into a single mutual inductance



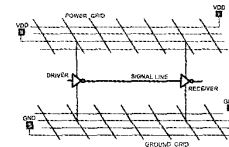
### Sparsification Issue

- ♦ Simply dropping small entries from the Partial Inductance Matrix, can lead to a non-positive definite matrix.



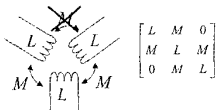
### Acceleration - PRIMA

- ♦ Passive Reduced-Order Interconnect Macromodeling Algorithm
  - [Odabasioglu - ICCAD '97]
  - Guaranteed passivity of reduced order model



### Sparsification Issue

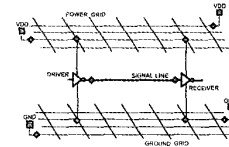
- ♦ Simply dropping small entries from the Partial Inductance Matrix, can lead to a non-positive definite matrix.
  - The incorrect sparsification results in system that generates energy.



### Acceleration - PRIMA

- ♦ Passive Reduced-Order Interconnect Macromodeling Algorithm
  - [Odabasioglu - ICCAD '97]
  - Guaranteed passivity of reduced order model

1. Define ports

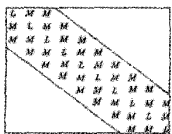


### Sparsification - Explicit

- ♦ Block diagonal sparsification [Gala DAC '00]
  - Guaranteed passive
  - Loss of accuracy



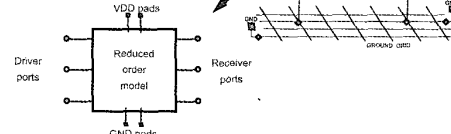
- ♦ Shift-and-truncate [Krauter ICCAD '95]
  - Remove couplings between wires with large separation
  - Better accuracy, guaranteed passive



### Acceleration - PRIMA

- ♦ Passive Reduced-Order Interconnect Macromodeling Algorithm
  - [Odabasioglu - ICCAD '97]
  - Guaranteed passivity of reduced order model

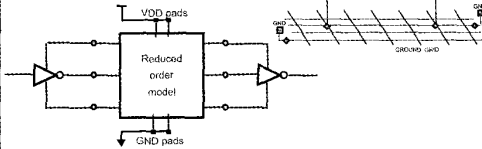
2. Compute multi-port reduced order model for the linear (RLC) circuit



### Acceleration - PRIMA

- Passive Reduced-Order Interconnect Macromodeling Algorithm
  - [Odabasioglu - ICCAD '97]
  - Guaranteed passivity of reduced order model

3. Combine with non-linear devices, power supply



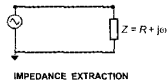
### Loop Inductance Approach

Use RL PEEC model and solve in frequency domain:

Step 1: Define grid, signal topology

Step 2: Define port at the driver end and short the receiver end of the signal.

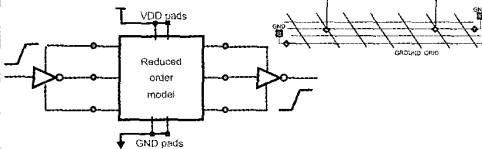
Step 3: Compute loop impedance



### Acceleration - PRIMA

- Passive Reduced-Order Interconnect Macromodeling Algorithm
  - [Odabasioglu - ICCAD '97]
  - Guaranteed passivity of reduced order model

4. Simulate the reduced system in SPICE



### Loop Inductance Approach

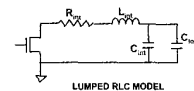
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Step 3: Compute loop impedance

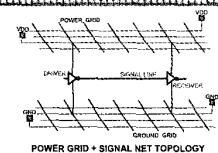
Step 4: Construct a lumped circuit model using the loop resistance and inductance with interconnect and load capacitance.



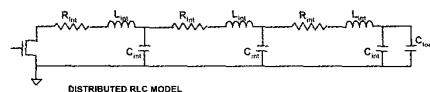
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Step 4: Construct a lumped circuit model using the loop resistance and inductance with interconnect and load capacitance.

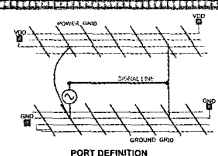
Can be extended to a distributed model

### Loop Inductance Approach

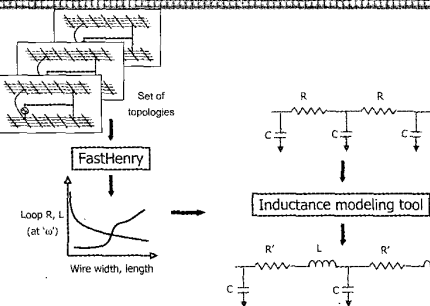
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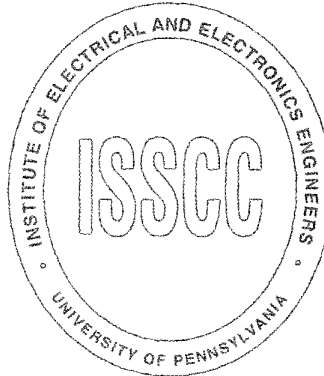
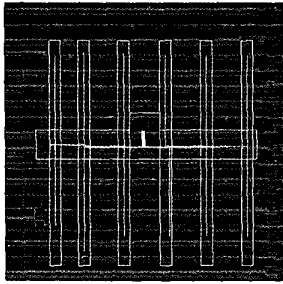
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### Characterization

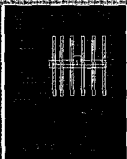


### Clock net topology



### PEEC vs. Loop Inductance

Clock Net 1



	ELEMENTS				DELAY (DIFF W.R.T. PEEC)	SKEW (DIFF W.R.T. PEEC)	RUN-TIME
	R	C	L	M			
PEEC	160k	400k	160k	8M	131ps	19ps	60min
RC	3k	6k	-	-	86ps (-45ps)	9ps (-10ps)	2min
LOOP	3k	6k	2k	-	116 (-15ps)	12ps (-7ps)	4min

### PEEC vs. Loop Inductance

Clock Net 2



	ELEMENTS				DELAY (DIFF W.R.T. PEEC)	RUN-TIME
	R	C	L	M		
PEEC	34k	70k	34k	400k	63ps	10min
RC	400	800	-	-	44ps (-19ps)	15s
LOOP	400	800	400	-	66ps (+3ps)	30s

### Summary

- ♦ On-chip inductance is important
  - Higher frequencies
  - Longer wires
- ♦ Modeling of inductance is difficult
  - Return paths complicated, frequency dependent
- ♦ Detailed model using PEEC
  - Complex
  - Run-time issues => Sparsification/Acceleration
- ♦ Simplified models using loop inductance
  - Fast
  - Accuracy issues => Use for specific topologies